AN APPROACH TO ROBUST MULTIATTRIBUTE CONCEPT SELECTION

Ashwin P. Gurnani  
Graduate Research Assistant  
University at Buffalo  
Department of Mechanical and Aerospace Engineering  
agurnani@eng.buffalo.edu

Tung-King See  
Graduate Research Assistant  
University at Buffalo  
Department of Mechanical and Aerospace Engineering  
tungsee@eng.buffalo.edu

Kemper Lewis  
Associate Professor  
University at Buffalo  
Department of Mechanical and Aerospace Engineering  
Corresponding Author: kelewis@eng.buffalo.edu  
(716) 645-2593 x2232

ABSTRACT

In this paper, we investigate and extend a method of selecting among a set of concepts or alternatives using multiple, potentially conflicting criteria. This method, called the Hypothetical Equivalents and Inequivalents Method (HEIM), has been shown to avoid the many pitfalls of already existing methods for such problems, such as pair-wise comparison, ranking methods, rating methods, and weighted sum approaches. The existence of multiple optimal sets of attribute weights based on a set of stated preferences is investigated. Using simple visualization techniques, we show that there is a range of weights that satisfy the constraints of HEIM. Depending on the attribute weights used, multiple possible alternative winners could exist. The visualization techniques, coupled with an indifference point analysis, are then used to understand the robustness of the solution obtained and determine the appropriate additional constraints necessary to identify a single robust optimal alternative.

1. INTRODUCTION

There is a growing recognition that engineering design can be classified as a set of various types of decisions, and the ability to make these decisions most effectively is the key to obtaining rational, logical, and optimal designs. The fundamental phases of Decision-Based Design (DBD) are determine possible design options and choose the best one [1-3]. Although the process of generating possible designs is not a trivial task and is an entire research area in itself [4], it is the second phase that is of interest to us in this paper.

Whenever we deal with decision-making, we face the need to make tradeoffs. We must pay more for a faster computer, expect existential problems while buying a used car, or wait in longer lines for higher airport security. More specifically, in engineering design, we can be certain that there is no one alternative that is best in every dimension. Therefore, how to make the “best” decision when choosing from among a set of alternatives in a design process has been a common problem in research and application in engineering design [5-11]. When the decision is multi-attribute in nature, common challenges include aggregating the criteria, rating of the alternatives, weighting of the attributes, and modeling strength of preferences in the attributes, among others, making it a very complex problem in itself.

Many effective decision-making tools have been developed in the past. With the increasing power of computers and the advent of tools such as virtual reality available on common desktops, visualization has become a common and effective tool for engineering design. The ability to visually represent a model, solution, or process has been utilized to make decisions that are more effective in the design of products and systems.

Drawings, sketches and drafting were the early forms of using visual imaging for engineering purposes. With the advent of modern technology, CAD/CAM/CAE systems are some of the graphical tools that are extensively used. In recent times, the ability to visualize 3-D space has been effectively used in design optimization, where feasible design ranges form the axes, and robust design regions are explored. Previous work in using visualization in design decision-making processes includes the use of visualization within the physical programming environment to monitor and interact with an
optimization process [11], the use of visualization strategies in nonlinear optimization [12], using a virtual reality environment for improved design decision-making [13-15] and Visual Design Steering (VDS) [16]. Moreover, visual tools have been developed to directly handle more than three dimensions in the design space, such as Graph Morphing [17], Cloud Visualization [18] and other novel multidimensional visualization techniques [11,19]. Although this paper does not develop a new visualization technique, it does utilize visualization techniques to gain insight into the behavior of a problem, and it is essential to realize the critical use of visualization in the engineering design and decision-making process.

In [20], various systematic methods of making multi-attribute design decisions such as pair-wise comparisons, ranking of alternatives, normalization, using strength of preferences with weighted sums and hypothetical equivalents have been described in detail with the use of an airplane selection problem. Additionally, the flaws of these methods have also been explained. In the next section, we review the method that has been developed, which aims at effectively making the decision of selecting among a set of alternatives based on conflicting, multiple attributes. In Section 3, we demonstrate the use of the method with an aircraft selection problem and present an updated approach to developing the hypothetical alternatives. In Section 4, we present visual images of the feasible sets of solutions obtained from the optimization problem developed in Section 3, provide insight into what the figures mean, and interpret the robustness of the feasible region. We then provide, in Section 5, some concluding remarks and topics for future work.

2. HYPOTHETICAL EQUIVALENTS AND INEQUIVALENTS METHOD

In [20], we introduce the concept of using Hypothetical Equivalents and Inequivalents in a multiattribute concept selection problem. In this paper, only a brief description of the Hypothetical Equivalents and Inequivalents Method (HEIM) is provided. The focus of this paper is incorporating an indifference point analysis and visualization routine to make the method more robust and applicable. HEIM has been developed to elicit stated preferences from a decision maker regarding a set of hypothetical alternatives in order to access attributes importance, and determine the weights directly from a decision maker’s stated preferences. The “equivalents” part of the method allows a decision maker to make statements like “hypothetical alternatives A and B are equivalent in value to me.” By making this kind of statement, a decision maker is identifying an indifference relationship between A and B. Indifference points have been used in previous work to determine attribute levels in order to solve for attribute weights [21]. However, unlike the previous work, we operate on hypothetical alternatives with given attribute levels directly.

Since the attribute levels of the given hypothetical alternatives can not be altered, finding hypothetical equivalents that are exactly of equivalent value to a decision maker, or “indifference points”, can be a challenging and time-consuming task [22], specifically in the context of constructing utility functions. Therefore, HEIM also accommodates inequivalents in the form of stated preferences such as “I prefer hypothetical alternative A over B.” When a preference is stated, by either equivalence or inequivalence, a constraint is formulated and an optimization problem is constructed to solve for the attribute weights. The weights are solved by formulating the following optimization problem,

\[
\text{Minimize } f(X) = \left(1 - \sum_{i=1}^{n} w_i \right)^2
\]

Subject to

\[
\begin{align*}
Xh & = 0 \\
g(X) & \leq 0
\end{align*}
\]

where, the objective function ensures that the sum of the weights is equal to one. \(X\) is the vector of attribute weights, \(n\) is the number of attributes, and \(w_i\) is the weight of attribute \(i\). The constraints are based on a set of stated preferences from the decision maker. The equality constraints are developed based on the stated preference of “I prefer alternatives A and B equally.” In other words, the value of these alternatives is equal, giving the following equation,

\[
V(A) = V(B) \quad \text{or} \quad V(A) - V(B) = 0
\]

The value of an alternative (alternative \(A\) in this case) is given as

\[
V(A) = \sum_{i=1}^{n} w_i a_i
\]

where \(a_i\) is the rating of alternative \(A\) on attribute \(i\). The inequality constraints are developed based on the stated preference of “I prefer A over B.” In other words, the value of alternative \(A\) is more than that of alternative \(B\), as shown in the following equations:

\[
V(A) > V(B) \quad \text{or} \quad V(B) - V(A) + \delta \leq 0
\]

where, \(\delta\) is a small positive number to ensure inequality of the values in Equation (4). The value of an alternative is given by Equation (3) as mentioned earlier.

While this method has been shown to avoid the theoretical pitfalls of the common decision making processes as discussed in [20], there are still significant research issues associated with applying the method to many types of multiattribute decisions in design. In Equation (1), we are trying to find a set of weights that sum to one and that satisfy the constraints. However, there could exist multiple sets of weights that satisfy these criteria. Therefore, this paper aims to investigate the presence of multiple solutions (if any) and their impacts on the chosen alternative.

In the next section, we systematically demonstrate how HEIM is to be used to solve a multiattribute decision problem and investigate if there exist more than one solution. In the latter half of the paper, we study the quality of the solution in terms of robustness by visualizing the feasible space for the decision problem.
3. MULTIATTRIBUTE DECISION MAKING USING HEIM

In this section, we use the same aircraft decision problem example used in [20] to demonstrate how HEIM creates multiple set of weights that are feasible and optimal. For illustration purposes, suppose a fictional airline carrier, Jetair, is planning to establish an air fleet to serve the routes on major cities among Asia Pacific countries and the United States. Jetair has decided to purchase only one type of aircraft for its entire fleet to reduce operating cost, similar to the strategy used by Southwest Airline and Jetblue Airways [23]. At this point, Jetair has identified four possible choices that meet its requirements and budget constraints. These choices are: Boeing 777-200 (Long Range), Boeing 747-200, Airbus 330-200 and Airbus 340-200.

3.1 Identify the Attributes

The first step is to identify the attributes that are relevant and important in the decision problem. The reason is that HEIM is not able to identify the absence of an important attribute. Techniques such as factor analysis [24] or value-focused thinking [25] can be used to identify the important/key attributes, reduce the attribute space, or eliminate unimportant or irrelevant variables/attributes. If an unimportant attribute is included in the process, HEIM will indicate the attribute’s limited role with a low weighing factor through the sequence of stated preferences over the hypothetical alternatives (e.g., the hypothetical alternatives that score well in important attributes will be preferred over those alternatives that score well in unimportant attributes). Also, by having unimportant attributes in the problem, the computational time of the method will increase. Therefore, identifying the key attributes is important to reduce the computational effort.

After reflecting upon the appeal of each of the four aircrafts, Jetair has identified three key attributes:

1) The number of passengers the plane can hold, which obviously reflects revenue for each flight.
2) The cruise range, where a longer cruise range will provide passengers with non-stop service.
3) The cruise speed, where a faster cruise speed means shorter times needed for each flight. Potentially, this could increase the frequency of turn-around times.

In Table 1, the data of the three attributes for the four aircrafts [26-27] are given.

<table>
<thead>
<tr>
<th>Aircraft</th>
<th>Speed (Mach)</th>
<th>Max. Cruise Range (nmi)</th>
<th>No. of Passengers</th>
</tr>
</thead>
<tbody>
<tr>
<td>B777-200</td>
<td>0.84</td>
<td>8820</td>
<td>301</td>
</tr>
<tr>
<td>B747-200</td>
<td>0.85</td>
<td>6900</td>
<td>366</td>
</tr>
<tr>
<td>A330-200</td>
<td>0.85</td>
<td>6650</td>
<td>253</td>
</tr>
<tr>
<td>A340-200</td>
<td>0.86</td>
<td>8000</td>
<td>239</td>
</tr>
</tbody>
</table>

Table 1 Attribute Data for Aircraft Alternatives

This problem is simplistic and is not realistic in terms of how airliners choose which aircraft to purchase. It is meant to illustrate the practical and theoretical advantages of using HEIM over other commonly used decision making methods to make selection decisions from among a set of alternatives in a multiattribute environment.

3.2 Determine the Strength of Preference within Each Attribute

Using a linear preference scale may not truly reflect a decision maker’s preferences. For instance, if Jetair prefers the increase in the number of passengers from 300 to 360 over the increase from 240 to 300 (because of the profit margin consideration), a linear preference function cannot be used to capture this preference structure. Jetair would be better off using a nonlinear strength of preference representation, better reflecting their true preferences. There are a number of ways to assess these strength of preferences, including utility theory methods [3,28,29]. In this paper, simple assumptions are made for illustration purposes. For the aircraft range, we assume an increase from 6,500 to 7,000 nmi is more preferred than an increase from 8,000 to 9,000 nmi (because an aircraft with cruise range less than 7,000 nmi, may have to make multiple stops for refueling). For the cruise speed, we assume a linear preference and for the number of passengers, we assume that an increase from 290 to 340 is slightly preferred over an increase from 240 to 290. These strength of preferences are shown in Figures 1-3. Table 2 shows the normalized numerical values for each attribute according to these strength of preference functions.

These strength of preference functions are based on the attribute range of the alternatives in the decision problem. If another alternative is added to the decision problem with an attribute value outside of the current range, then the strength of preference functions must be formulated and normalized again. For instance, in Figure 2, the lowest and highest cruise ranges, 6650 and 8820 nmi, are used to formulate the preference score. If another alternative with a cruise range lower than 6650 nmi or higher than 8820 nmi is added to the decision problem, the strength of preference function must be reformulated using the new upper and lower cruise ranges.
To use HEIM, setting up the hypothetical alternatives is next important step. The purpose of this step is to establish a set of hypothetical alternatives that a designer feels indifferent between or that a designer can differentiate if one alternative is preferred over the other. This is done so that the preference structure can be modeled using not only equality equations, but also inequality equations. Therefore, the set of preference weights (design variables) can then be solved by using optimization techniques.

In [20], the hypothetical alternatives were developed by simply mixing the upper and lower bounds of each attribute in different combinations. However, a more systematic approach is needed to develop the hypothetical alternative so as to efficiently sample the design space. In this paper, we use a fractional factorial experimental design [30]. However, other effective experimental designs such as Central Composite Design [31] could also be used. A three factor, three level fractional factorial design is used since there are three design variables (one weight for each of three alternatives) each with three levels in this decision problem. Table 3 shows the experimental design with the following corresponding strength of preference score:

- Level 1: Score of 0
- Level 2: Score of 50
- Level 3: Score of 100

Table 4 shows the corresponding numerical values for each attribute with the consideration of the strength of preference scores as shown in Section 3.2 (Figures 1-3). For example, level 2 for Factor 2, Range, corresponds to a strength of preference score of 50, which in turn corresponds to a attribute value of 6900 nmi, as shown in Figure 2.

3.4 Normalize the Scale and Calculate the Value for Each Alternative

Normalization is required to eliminate the dimensions from the problem. However, normalization can be carried out only after the preference strengths have been determined in order to avoid the flaws of assuming a linear preference structure. In addition, the values of each alternative as a function of the attribute weights are also calculated and are used in the optimization problem in the next section. The attribute data and the normalized scores and value equations for the hypothetical alternatives are shown in Table 4 and 5, respectively.
Table 5 Normalized Score for Table 4

### 3.5 Formulate the Preference Structure as an Optimization Problem

To apply optimization techniques in HEIM, the preference structure is formulated into an optimization problem. The preferences structure is identified based on the hypothetical alternatives in Table 4.

Assume that Jetair believes rating nine alternatives at once is difficult and therefore, they rate three alternatives at a time. For the first three alternatives, Jetair has the preference structure as \( C \succ B \succ A \), where \( \succ \) indicates, “preferred to”. From this first set of preference, two non-redundant constraints can be generated, \( C \succ B \) and \( B \succ A \). By using the values shown in Table 5, the constraints can be written as,

\[
G_1 = -0.5w_1 - 0.5w_2 + 0.5w_3 + \delta \leq 0 \\
G_2 = -0.5w_1 - 0.5w_2 - w_3 + \delta \leq 0
\]

where the \( \delta \) is 0.001 that is sufficient to ensure inequality of the value. For the remaining two sets of alternatives, the preference structures by Jetair are, \( F \succ E \succ D \) and \( G \succ I \succ H \). Therefore, the complete optimization problem for this example is shown in Equation (6).

\[
\text{Min } F = \left[ 1 - (w_1 + w_2 + w_3) \right]^2
\]

Subject to,

\[
G_1 = -0.5w_1 - 0.5w_2 + 0.5w_3 + \delta \leq 0 \\
G_2 = 0.5w_1 - 0.5w_2 + 0.5w_3 + \delta \leq 0 \\
G_3 = -0.5w_1 + w_2 - w_3 + \delta \leq 0 \\
G_4 = w_1 - 0.5w_2 - w_3 + \delta \leq 0 \\
G_5 = -0.5w_1 - 0.5w_2 + 0.5w_3 + \delta \leq 0 \\
G_6 = 0.5w_1 - 0.5w_2 + 0.5w_3 + \delta \leq 0
\]

Side Constraints: \( 0 \leq w_i \leq 1 \)

Note that \( G_4 \) and \( G_6 \) are redundant constraints (they are the same as \( G_1 \)). In the computational stage, these two redundant constraints are not included. Current research work is focused on identifying how many constraints are necessary (determining over and under constrained problems) and eliminating redundant preference assessments in order to make the computational steps more efficient.

### 3.6 Solve for the Preference Weights

Solution for the preference weights can be obtained using any optimization technique. However, since the constraints are linear, Sequential Linear Programming (SLP) or Generalized Reduced Gradient (GRG) methods work well [32]. Using SLP, and given a single starting point, one feasible solution set of weights is \( [0.33, 0.33, 0.33] \).

### 3.7 Make Decision

With the attribute weights from the previous section, a weighted sum result is shown in Table 6. The preferred aircraft is B747.

Since it is assumed that a linear combination of attributes represents the value of an alternative (Equation 3), and because the domain of choices is discrete, many of the noted pitfalls of weighted-sum approaches are avoided [33-35]. In other words, new alternatives are not searched for and developed outside of those in Table 6. However, the sensitivity of the best alternative to changes in the weights is important, as the following discussion illustrates.

Because the weights were found using their sum as an objective function, there may be many possible sets of weights whose sum equals one and that satisfy the constraints from the stated preferences. Using another starting point to solve the optimization problem using SLP, a different set of weights is found \( [0.4,0.3,0.3] \). The modified weighted sum results for this set of weights is shown in Table 7.

As Table 7 indicates, the A340 is now the winning alternative with the highest score. This indicates that using the preference structure and resulting constraints in Equation (6), more than one winning alternative can be found. This is obviously not a desirable state. Since the winning alternative is not robust (it
can change depending upon starting point, etc.), it would indicate a need to investigate the presence of multiple solutions to Equation (6). In fact, it would indicate that Equation (6) is an underconstrained problem. If more constraints were added, perhaps the robustness of the solution would increase and the winning alternative would not change across multiple sets of feasible weights. This is precisely the issue that we investigate in the next section using visualization techniques and indifference points.

### 4. VISUALIZATION AND ROBUSTNESS STUDY

In the previous section, we recognized that by using HEIM, we are able to determine a solution for the weights such that we could rank the various alternatives according to the total score obtained. However, since the objective function used is merely a constraint requiring that the weights sum to one, we also showed there may exist more solutions that were both optimal (sum is equal to one) and feasible (satisfy the preference constraints).

With the potential for more than one set of feasible and optimal weights existing, it is also possible that within the feasible set, not all weight values will result in the same alternative having the highest value. Indeed, in the previous section, it was shown that with the simple aircraft example and six preference constraints, multiple aircraft could be deemed to be the preferred aircraft.

Since it is possible for multiple alternatives to be the preferred solution to the selection problem, one of the goals of this work is to further develop the HEIM method in order to identify one robust solution. The classical definition of “robust” is a solution that is insensitive to variations in control and noise factors [30]. The term “robust” in the context of this paper implies a preferred alternative that is insensitive to different sets of feasible weights. As shown in the previous section in Tables 6 and 7, neither solution is robust, as a slight change in attribute weights results in a different preferred aircraft. To achieve a robust solution to the multiattribute decision-making problem, an extension of the HEIM method to reduce the feasible region to one winning alternative is presented.

**Step 1:** Generate points (combinations of weights) in the feasible space, as defined by the preference constraints and the constraint that the sum of the weights must equal one. A grid or random point generation technique can be used to generate the points.

**Step 2:** Evaluate the value functions for all alternatives at each point and determine the winning alternative for that point. In general, k possible winning alternatives could exist.

**Step 3:** Where k is greater than one (more than one alternative is determined to be the winner), equate the value functions of all winning alternatives, taken two at a time. Thus, set

\[
V(A_i) = V(A_k) \quad i \neq k, 1, m = 1, k
\]

Since we are using a weighted sum approach to determine the solution, we represent the values of the winning alternatives as,

\[
V(A_i) = w_1 + w_2 + \ldots + w_n
\]

where \( w_1, w_2, \ldots, w_n \) correspond to the normalized attribute ratings for alternative \( A_i \) and \( A_k \), and \( w_1, w_2, \ldots, w_n \) correspond to the attribute weights. Thus, from Equations (7) and (8), we get

\[
(a_i - a_m)w_1 + (a_i - a_m)w_2 + \ldots + (a_i - a_m)w_n = 0
\]

where,

\[
d_i = a_m - a_i \quad i = 1, n
\]

**Step 4:** Develop new hypothetical alternatives from Equation (9) and elicit a stated preference between them. To accomplish this, the \( d_i \)'s are divided into two parts such that two new hypothetical alternatives are generated. Thus, let

\[
d_i = h_1 + h_2 \quad i = 1, n
\]

Then, we get two new hypothetical alternatives – \( h_1 \), with normalized attribute values \( h_{11}, h_{12}, \ldots, h_{1n} \), and \( h_2 \), with normalized attribute values \( h_{21}, h_{22}, \ldots, h_{2n} \). The value functions of \( h_1 \) and \( h_2 \) are:

\[
V(h_1) = h_{11}w_1 + h_{12}w_2 + \ldots + h_{1n}w_n
\]

\[
V(h_2) = h_{21}w_1 + h_{22}w_2 + \ldots + h_{2n}w_n
\]

Note that the attribute values for the hypothetical alternatives (e.g., \( h_{11}, h_{12}, \ldots, h_{1n} \) generated above are normalized. Once converted to the actual attribute levels, the preference of the decision maker is stated between \( h_1 \) and \( h_2 \).

We use the new hypothetical alternatives to determine the preference of the decision maker to determine additional constraints similar to the procedure explained in Section 3.5. In general, hypothetical alternatives are used in order to explore the alternative space in a controlled and deliberate manner to capture a decision maker’s preferences over a range of attribute values. Also, hypothetical alternatives allow the decision maker to state his or her preferences without explicitly showing bias towards one of the actual alternatives.

**Step 5:** Using the actual attribute levels of the new hypothetical alternatives, the decision maker decides if \( h_1, h_2, h_3, \ldots, h_n \) generated above are normalized. Once converted to the actual attribute levels, the preference of the decision maker is stated between \( h_1 \) and \( h_2 \).

Steps 3 through 5 are repeated for all winning alternatives to determine a robust feasible region where only one alternative is the winner.
To demonstrate the above steps for our aircraft selection example, we use the aid of visualization. Since we have three attributes, we can use a 3-D world to show the feasible region, constraints, and winning alternatives. Color coding is used to indicate the different winning alternatives and to help visualize the effect of the additional constraints on obtaining a robust solution.

To visualize the optimization problem and identify the feasible space, we used the OpenGL Programming API [36] to plot the three attribute weights, one on each axis. Additionally, each weight is restricted to have values from zero to one, as side constraints on the three variables $w_1$, $w_2$, and $w_3$. In Figure 4, the 3-D space is shown along with the outline of the plane representing the constraint in Equation (12), marked in red.

$$w_1 + w_2 + w_3 = 1.0$$  \hspace{1cm} (12)

**Figure 4 3-D Space Visualization with $\sum_{i=1}^{3} w_i = 1$ Plane**

*Step 1:* Generate points – We randomly sample weight vectors $(w_1, w_2, w_3)$ and determine those that satisfy Equation (12).

*Step 2:* Evaluate value functions – Next, we evaluate the value functions of the four alternatives for each of the generated points using the value functions listed in Table 8. Note that the remaining constraints in Equation (6) are not yet considered.

<table>
<thead>
<tr>
<th>Aircraft</th>
<th>Speed $(w_1)$</th>
<th>Max. Range $(w_2)$</th>
<th>No. of Passengers $(w_3)$</th>
<th>Value Functions</th>
</tr>
</thead>
<tbody>
<tr>
<td>B777-200</td>
<td>0</td>
<td>100</td>
<td>35</td>
<td>$w_2 + 0.35w_3$</td>
</tr>
<tr>
<td>B747-200</td>
<td>35</td>
<td>50</td>
<td>100</td>
<td>$0.35w_1 + 0.5w_2 + w_3$</td>
</tr>
<tr>
<td>A330-200</td>
<td>35</td>
<td>0</td>
<td>5</td>
<td>$0.35w_1 + 0.05w_3$</td>
</tr>
<tr>
<td>A340-200</td>
<td>100</td>
<td>80</td>
<td>0</td>
<td>$w_1 + 0.8w_2$</td>
</tr>
</tbody>
</table>

**Table 8 Value Functions of the Alternatives Based on Preferences From Table 2**

Once the value functions are evaluated, the alternative with the highest value for each of the generated weight vectors is determined and represented with a particular color. Table 9 lists the color corresponding to each alternative.

<table>
<thead>
<tr>
<th>Alternative</th>
<th>Color of Point</th>
</tr>
</thead>
<tbody>
<tr>
<td>B777</td>
<td>Orange</td>
</tr>
<tr>
<td>B747</td>
<td>Blue</td>
</tr>
<tr>
<td>A330</td>
<td>Black</td>
</tr>
<tr>
<td>A340</td>
<td>Green</td>
</tr>
</tbody>
</table>

**Table 9 Color Code to Represent Various Alternative Winners**

Figure 5 indicates the initial feasible space (points satisfying Equation (12)) and shows the different regions of winning alternatives.

As can be seen in Figure 5, the blue points dominate the feasible region, which implies that the B747 (blue points) has the greatest possibility of being the alternative of choice. However, depending on the constraint developed, the B777 (orange points) or the A340 (green points) can also be selected.

As mentioned earlier, the space in Figure 5 does not include any of the preference structure constraints from Equation (6). Ideally, as these constraints are included, the feasible space in Figure 5 will shrink. Next, we include these constraints specified in Equation (6). The constraint corresponding to the stated preference between the first two hypothetical alternatives, shown as Equation (13), is included in determining the new feasible space. Figure 6 shows the feasible space and alternative winners.

$$G_1 = -0.5w_1 - 0.5w_2 + 0.5w_3 + \delta \leq 0 \hspace{1cm} (13)$$

Note that Equation (13) has been defined in Equation (6).
Figure 6 Preferred Alternatives with One Preference Constraint

Figure 6 shows a clear decrease in the region spanned by the blue points, while the region spanned by the green points has remained the same. This result would lead us to expect the A340, which corresponds to the green color, to be the winning alternative most of the time. However, it would be a non-robust solution, as three winning alternatives are still possible. In Figure 7, the remaining constraints given in Equation (6) are included, alternative value functions are evaluated, and the alternative with the highest value is represented with the color codes of Table 9. Also shown are the two different solutions from Section 3.7 clearly indicating the different preferred solutions.

Figure 7 Final Feasible Space Including all Constraints Specified in Equation (6)

Figure 7 indicates that the solution is still not sufficiently constrained to produce a robust alternative winner. The reason is that both blue and green points exist in the feasible space, which represent the B747 and the A340 as the winning alternatives, respectively. However, it is now clear as to why two different winning alternatives are found in Section 3.7. We now follow steps 3 through 5 given below to determine a robust winning alternative.

Step 3: Equate the value functions of multiple winning alternatives – Now that we have represented the feasible space and realized we do not have a single winning alternative, we need to determine more constraints such that the optimization problem always gives one robust winner. To do this, we first need to determine the line separating the region of blue and green points in Figure 7. If this line can be mathematically determined and converted into a preference constraint, then one side of the line could be deemed infeasible, eliminating either the green or blue regions from consideration. This dividing line is the line of indifference between the blue and green alternatives because any combination of weight values on this line will give the same overall score for both alternatives. Therefore, equating the value functions of the green alternative (B747) and the blue alternative (A340) from Table 8, we get,

\[ V(B747) = V(A340) \]
\[ 0.35w_1 + 0.5w_2 + w_3 = w_1 + 0.8w_2 \]  
\[ -0.65w_1 - 0.3w_2 + w_3 = 0 \]  

Step 4: Determine the hypothetical alternatives – Once we have the equation of the indifference line, we rearrange the terms to determine two different value functions, which are then converted into two new hypothetical alternatives. Rearranging the terms in Equation (14), we get,

\[ 0.35w_1 + 0.7w_2 + w_3 = w_1 + w_2 \]  

Note that Equation (15) is only one of the possible rearrangements of Equation (14). The expressions on the right and left of Equation (15) correspond to two different value functions, and we can construct two new hypothetical equivalents, one from each side of the equation. The normalized values of these hypothetical alternatives – the coefficients of the weights in Equation (15) – are shown in Table 10.

<table>
<thead>
<tr>
<th>Alternative</th>
<th>Speed</th>
<th>Range</th>
<th>Passengers</th>
</tr>
</thead>
<tbody>
<tr>
<td>J</td>
<td>0.35</td>
<td>0.7</td>
<td>1</td>
</tr>
<tr>
<td>K</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 10 Normalized Attributes of Two New Hypothetical Alternatives From Equation (11)

The actual values for the cruise speed, range, and number of passengers for alternatives J and K are specified in Table 11. These values are obtained using Table 10 and the strengths of preference charts shown in Figures 1-3.

<table>
<thead>
<tr>
<th>Alternative</th>
<th>Speed</th>
<th>Range</th>
<th>Passengers</th>
</tr>
</thead>
<tbody>
<tr>
<td>J</td>
<td>0.847</td>
<td>7620</td>
<td>366</td>
</tr>
<tr>
<td>K</td>
<td>0.86</td>
<td>8820</td>
<td>235</td>
</tr>
</tbody>
</table>

Table 11 Actual Attribute Values of New Hypothetical Alternatives

Step 5: State preferences for hypothetical alternatives – Now that we have developed two new hypothetical alternatives, the decision maker states his preference between alternatives J and K. If the decision maker prefers J over K, we get,

Alternative J > Alternative K
\[ 0.35w_1 + 0.7w_2 + w_3 > w_1 + w_2 \]  

Equation (16) provides the extra constraint needed to achieve a single robust winner. This constraint is incorporated into our 3-D world, and the result is shown in Figure 8.
Figure 8 shows a complete blue region, implying that the winning alternative is always the B747. Thus, Equation (16) provided the necessary constraint to obtain a completely robust solution. If, however, the decision maker reversed his preference, that is, he preferred alternative K to alternative J, we get,

Alternative J < Alternative K  
0.35w₁ + 0.7w₂ + w₃ < w₁ + w₂  (17)

Incorporating Equation (17), we get a feasible region with only green solutions, as shown in Figure 9.

From Figures 8 and 9, we conclude that we now have a sufficiently constrained feasible region with the help of the hypothetical alternatives in Table 11. The hypothetical alternatives make it easy for the decision maker to state his or her preference, especially where he or she does not have information on the actual value functions of the alternatives. Moreover, they do not need to quantify how much they prefer one alternative to the other. All that the decision maker needs to specify is their individual preference of one alternative to the other. The solution to the optimization problem then gives the values of the weights for the attributes, which quantify the relative importance among the various attributes.

5. CONCLUSIONS AND FUTURE WORK

In this paper, we have used a simple example of the selection of an airplane to demonstrate the Hypothetical Equivalents and Inequivalents Method. We have investigated the presence of multiple solutions and their impact on the alternative chosen. We also formulated an approach to determine a single robust winning alternative by generating hypothetical alternatives based on equating the value functions of multiple winning alternatives.

With the use of simple visualization techniques, it was possible to represent the feasible region, including the most preferred alternative within the feasible region. When more than one preferred alternative is possible, indifference points are used to construct additional hypothetical alternatives to elicit the necessary preferences to reduce the feasible region appropriately. With the additional stated preference, we showed that a robust region of solutions could be identified that results in only one preferred alternative. Expanding the Hypothetical Equivalents and Inequivalents Method using visualization techniques and concepts from underconstrained programming is the primary contribution of this work. The outcome is an approach to ensure that enough preference constraints are elicited to identify one preferred alternative across the entire feasible region. By using visualization, we can strategically identify the most effective additional constraints necessary to reduce the feasible region until one alternative is found.

Future work includes extending the use of visualization to more than three dimensions. If more than three attributes are present, concepts from n-dimensional representation and visualization will be necessary to aid the process of developing new constraints. Another area of future work is to expand HEIM to handle group decision-making and uncertainty that might exist within attribute values. Issues of intransitivity and preference aggregation will be the focus of this work.

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