An Investigation of Equilibrium Stability in Decentralized Design Using Nonlinear Control Theory

Vincent Chanron∗, Tarunraj Singh† and Kemper Lewis†

University at Buffalo, SUNY, Buffalo, NY 14260, USA

The focus of this paper is a theoretical study of the design of complex engineering systems. More particularly, this paper studies the stability of equilibriums in decentralized design environments. Indeed the decomposition and coordination of decisions in the design of complex engineering systems is a great challenge. Companies who design these systems routinely allocate design responsibility of the various subsystems and components to different people, teams or even suppliers. The mechanisms behind this network of decentralized design decisions create difficult management and coordination issues. However, developing efficient design processes is paramount, especially with market pressures and customer expectations. Standard techniques to modeling and solving decentralized design problems typically fail to understand the underlying dynamics of the decentralized processes and therefore result in suboptimal solutions. This paper aims to model and understand the mechanisms and dynamics behind a decentralized set of decisions within a complex design process.

Complex systems that are multidisciplinary and highly nonlinear in nature are the primary focus of this paper. Therefore, techniques such as Response Surface Approximations and Game Theory are used to discuss and solve the issues related to multidisciplinary optimization, while techniques and knowledge from Nonlinear Control theory are used to discuss the stability of some equilibrium points. Illustrations of the results are provided in the form of the study of a decentralized design of a pressure vessel.

Keywords: Decentralized Design, Decomposition, Game Theory, Nash Equilibrium, Response Surface, Nonlinear Control, Lyapunov Theory.

I. Introduction

The focus of this paper is the design of complex engineering systems, or those systems that necessitate the decomposition of the system into smaller subsystems in order to reduce the complexity of the design problems. Most of these systems are very large and multidisciplinary in nature, and therefore have a great number of subsystems and components.

Over the past years, a lot of different techniques, algorithms, software and methods have been proposed to solve these design problems. They mainly concentrate on finding a solution which is optimal in some sense, while keeping the computational cost as low as possible. One of these methods, known as decomposition, is now seen as a necessary step in design. Indeed, those complex engineering systems are multidisciplinary in nature, and it is therefore impossible for one designer, or even a single design team, to consider the entire system as a single design problem. Typically, in complex systems, breaking it up into smaller units or subsystems will make the system more manageable.1,2

The decentralization of decisions is unavoidable in a large organization where having only one centralized decision maker is usually not applicable.3 A more effective way is to delegate decision responsibilities to the appropriate person, team or supplier. In fact, decentralization is recommended as a way to speed up product development processes and decrease the computational time and the complexity of the problem.4

∗Graduate Research Assistant, Department of Mechanical and Aerospace Engineering, University at Buffalo.
†Associate Professor, Department of Mechanical and Aerospace Engineering, University at Buffalo, senior member AIAA.
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While the decomposition of complex problems certainly creates a series of smaller, less complex problems, it also creates several challenging issues associated with the coordination of these less complex problems. The origin of these problems is the fact that the less complex subproblems are usually coupled and dependent upon information from other subproblems. The ideal case would be when a system could be broken up into subsystems without interdependence. Unfortunately, there are usually design variables and parameters that have an influence on several subproblems. A formal definition of coupled subsystems can be found in.\textsuperscript{5}

Previous work has been done on the decomposition of the system into smaller ones; using Design Structure Matrices,\textsuperscript{6} a hierarchical approach,\textsuperscript{7} or by effectively propagating the desirable top level design specifications to appropriate subsystems,\textsuperscript{8} and their efficiency has also been compared.\textsuperscript{9}

Also previous work has concentrated on solving those design problems with interacting subsystems using Game Theory. The main goal is to try to improve the quality of the final solution in a multiobjective, distributed design optimization problem.\textsuperscript{10} Previous work in Game Theory includes work to model the interactions between the designers if several design variables are shared among designers.\textsuperscript{11} In,\textsuperscript{12} Game Theory is formally presented as a method to help designers make strategic decisions in a scientific way. In,\textsuperscript{13} distributed collaborative design is viewed as a non-cooperative game, and maintenance considerations are introduced into a design problem using concepts from Game Theory. In,\textsuperscript{14} the manufacturability of multiagent process planning systems is studied using Game Theory concepts. In,\textsuperscript{15} non-cooperative protocols are studied and the application of Stackelberg leader/follower solutions is shown. Also in,\textsuperscript{16} a Game Theory approach is used to address and describe a multifunctional team approach for concurrent parametric design.

This set of previous work has established a solid foundation for the application of game theory in design, but has not directly studied the mechanisms of convergence in a generic decentralized design problem.

This paper does not propose any other decomposition method, nor another Game-Theoretic approach to the design process. However, it tries to formally describe the dynamics and interactions involved in such design scenarios. We believe that, in order to be able to design better, those dynamics have to be well understood. They will be a strong basis for further research in this area. The novelty of this work stands in the fact that concepts from Nonlinear Control Theory are applied to the design scenarios in order to study the stability of some highly nonlinear equilibriums. The next section describes how we propose to look at those scenarios using those new techniques.

II. Design Scenarios

In this section, the main game theory scenarios used to solve large multiobjective design problems are reviewed and discussed. We assume that the design problem has already been subdivided into smaller subsystems, either naturally because several different companies interact on the design of the same product, or either because the system has been subdivided into smaller subsystems using one of the techniques described in the previous section. A good description of the different scenarios in design can be found in\textsuperscript{15} and\textsuperscript{17}

As mentioned in the previous section, Game Theory is usually used as a way to study those design scenarios. Table 1 presents the Game-Theoretic formulation for an optimization design problem with two designers (also called players). In this table, \( x_1 \) represents the vector of design variables controlled by designer 1, while designer 2 controls design variable vector \( x_2 \). We denote \( x_{1c} \) and \( x_{2c} \) the nonlocal design variables, variables that appear in a model but are controlled by the other player.

<table>
<thead>
<tr>
<th>Player 1’s Model:</th>
<th>Player 2’s Model:</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F_1(x_1, x_{2c}) = { F_1^1, F_1^2, ..., F_1^m } )</td>
<td>( F_2(x_2, x_{1c}) = { F_2^1, F_2^2, ..., F_2^l } )</td>
</tr>
<tr>
<td>\textit{subject to}</td>
<td>\textit{subject to}</td>
</tr>
<tr>
<td>( g_1^j(x_1, x_{2c}) \leq 0, \quad j = 1..m_1 )</td>
<td>( g_2^j(x_2, x_{1c}) \leq 0, \quad j = 1..m_2 )</td>
</tr>
<tr>
<td>( h_1^k(x_1, x_{2c}) = 0, \quad k = 1..l_1 )</td>
<td>( h_2^k(x_2, x_{1c}) = 0, \quad k = 1..l_2 )</td>
</tr>
<tr>
<td>( x_{1L} \leq x_1 \leq x_{1U} )</td>
<td>( x_{2L} \leq x_2 \leq x_{2U} )</td>
</tr>
</tbody>
</table>
A complete description of all the protocols can be found in, but we present here only the three main types.

**Cooperative Protocol**

In this protocol, both players have knowledge of the other player’s information and they work together to find a Pareto solution. A pair \((x_1, x_2)\) is Pareto optimal\(^{19}\) if no other pair \((x_1', x_2')\) exists such that

\[
F_i(x_1, x_2) \leq F_i(x_1', x_2') \quad i = 1, 2
\]

\[
& F_j(x_1, x_2) < F_j(x_1', x_2') \quad \text{for at least one } j = 1, 2
\]

(1)

**Noncooperative Protocol**

This protocol occurs when full coalition among players is not possible due to organizational, information, or process barriers. Players must make decisions by assuming the choices of the other decision makers. In an iterative approach, the final solution would be a Nash equilibrium. A strategy pair \((x_{1N}, x_{2N})\) is a Nash solution if

\[
F_1(x_{1N}, x_{2N}) = \min_{x_1} F_1(x_1, x_{2N})
\]

\[
\text{and} \quad F_2(x_{1N}, x_{2N}) = \min_{x_2} F_2(x_{1N}, x_2)
\]

(2)

This solution has the property of being individually stable, but is not necessarily collectively stable. The Nash Equilibrium also has the property of being the fixed point of two subsets of the feasible space:

\[(x_{1N}, x_{2N}) \in X_{1N}(x_{2N}) \times X_{2N}(x_{1N})\]

where

\[X_{1N}(x_2) = \{x_{1N} | F_1(x_{1N}, x_2) = \min_{x_1} F_1(x_1, x_2)\}\]

\[X_{2N}(x_1) = \{x_{2N} | F_2(x_1, x_{2N}) = \min_{x_2} F_2(x_1, x_2)\}\]

are called the Rational Reaction Sets of the two players. The Rational Reaction Set (RRS) of a player is a function that embodies his reactions to decisions made by other players.

**Leader/Follower Protocol**

When one players dominates another, they have a leader/follower relationship. This is a common occurrence in a design process when one discipline dominates the design (when one discipline plays a large role), or in a design process that involves a sequential execution of interrelated disciplinary processes. P1 is said to be the leader if he/she declares his/her strategy first, by assuming that P2 behaves rationally. Thus the model of P1 as a leader is the following

\[
\text{Minimize} \quad F_1(x_1, x_2)
\]

\[
\text{subject to} \quad x_2 \in X_{2N}(x_1)
\]

(3)

where \(X_{2N}(x_1)\) is the RRS of player 2.

The focus of this paper is design in decentralized environments. In that case, even within the same corporation, perfect information and cooperation is difficult to achieve due to several factors, including the complexity of the design, geographic separation or information privacy. Therefore, we focus on noncooperative relationships between designers. In other words, we focus on decentralized design scenarios where full and efficient exchange of all information among subsystems is not possible.

In these environments, the goal is to determine whether the Nash equilibrium is collectively stable or not. This issue has never been investigated although its relevance is noted in several papers.\(^{10,20}\) This issue of an unstable equilibrium is challenging. Indeed, in the case of instability, designers will never agree on a final design since one of the designers will always be able to change the value of their design variables and improve their objective function. In this case, the process by which the two designers might go about choosing the final design is then difficult to predict, but in the absence of any additional information or intervention by a third party, it seems obvious that choosing the final design will be problematic. The first steps towards the study of stability have been laid out for simple design problems with quadratic objective functions.\(^{21}\) The focus of this paper is to study those properties for more complicated and nonlinear problems. The next section presents the steps to follow for such a study.
III. Equilibriums of the design space

In this section, we describe how to obtain the equilibrium points of a specified design space, with objectives, constraints and designers. As mentioned in the Noncooperative Protocol in the previous section, the solution of a noncooperative design scenario is the possible intersection of the Rational Reaction Sets of every designer. For highly nonlinear problems however, the exact equations of the RRS is seldom possible to obtain, and approximating functions have to be used. The most commonly used approximating functions are polynomial response surface equations. To be used, the design space of each designer has to be sampled: this is traditionally done using statistical experimentation (design of experiments - DOE). A conceptual outline for the construction of the RRS is illustrated in Figure 1.

To approximate the RRS for Player 1, for example, the following procedure is carried out. First, different points are sampled in Players 2’s nonlinear design space according to a specific DOE protocol (e.g. central composite design, full factorial, partial factorial). At each of those specified points, Player 1’s model is solved. Once these steps are completed, a second order polynomial (in our case) is fitted to relate $x_2$ to $x_1$.

Those Response Surface equations embody the strategies of the various designers, and their possible intersection represents the different equilibriums of the design space.

Once again, these equilibriums, known as Nash solution points, are individually stable but not necessarily collectively stable, and it is the aim of this paper to introduce methodologies on how to investigate these properties. The next section presents several methods, derived from Nonlinear Control theory, that can be used to determine the stability of equilibriums.

IV. Stability of equilibriums

The focus of Control theory is the analysis and the design of control systems. The analysis part consists of the determination of the characteristics of a given system’s behavior, one of which being its equilibrium points and their stability. Techniques from this field can therefore be applied to the study of equilibriums in engineering design, after making some changes to adapt them to this kind of problems. Besides, since the focus of this paper is highly nonlinear design problem, the techniques that are used, and described in this section are derived from Nonlinear Control theory.

The previous section describes how to get approximations for the RRS of each player involved in the design process. In a sequential approach to design, information is exchanged back and forth between the designers before reaching a final agreement. This can be compared to a discrete time control problem, similar to the time series formulation presented in. The most general discrete time update equation is shown in Equation (4).

$$x(k + 1) = f(x(k))$$ (4)

where $f$ is a nonlinear function of the states at the previous time steps, and where $x$ is the state vector made of all the design variables of every designer. The function $f$ is defined easily from the equations of the Response Surface approximations found using the method described in Figure 1. $f$ is an autonomous
function since it does not depend explicitly on time. Equation (4) is the key equation that is used with every technique presented next.

1. Finding the equilibrium points

As discussed earlier, the equilibrium points of the design space lie at the intersection of the Response Surfaces approximating the RRS of every designer. This correspond to a point where the control system studied has reached a steady-state. Mathematically, one can solve for these steady-state points by setting \( x(k+1) = x(k) \) in Equation (4). Solving this equation will give us the set of equilibrium points. This set can be empty (there is therefore no Nash solution), or can have one or several equilibriums, whose stability need to be established. To do so, for every equilibrium studied, the considered equilibrium point needs to be moved back as the origin of the update equation (4) by a simple change of variable. Therefore, the following techniques can now be applied to the new update equation to study the stability of the origin, which is the equilibrium point considered.

However, we first need to define stability, as used in Nonlinear Control Theory. The definition of stability in the sense of Lyapunov is given next.

**Stability:** The equilibrium state \( x = 0 \) is said to be stable if, for any \( R > 0 \), there exists \( r > 0 \), such that if \( ||x(0)|| < r \), then \( ||x(k)|| < R \) for all \( k \geq 0 \). Otherwise, the equilibrium point is unstable.

Lyapunov stability is definitely an interesting property for our problem. However, another relevant information towards the study of the collective stability of the Nash equilibriums would be to know the properties of the region around the equilibrium. In other words, a valuable information would be to know the domain of attraction of the equilibrium.

**Domain of attraction:** Set of all points such that trajectories initiated at these points eventually converge to the origin.

This also introduces the concept of asymptotic stability which requires stability and the convergence of the states to 0.

2. Linearization and Local Stability

Lyapunov’s linearization method is concerned with the local stability of the equilibrium of a nonlinear systems. It is the formalization of the intuition that a nonlinear system should behave similarly to its linearized approximation in a small range around the equilibrium. To do so, only the linear terms of the update Equation (4) are kept. Equation (5) shows the linearization of the original nonlinear system at the equilibrium point 0.

\[
x(k+1) = Ax(k) \quad (5)
\]

This equation is of course similar to the update equation of a linear system, for which the study of stability is known. The stability of equilibriums for linear decentralized design problems can be found in.\(^{21,25}\)

We recommend the linearization to be the first step to check local convergence properties around the equilibrium. Indeed, the stability of the linearized system gives us interesting insights on the initial nonlinear system thanks to the theorem of the Lyapunov’s linearization method.

- If the linearized system is strictly stable, then the origin is asymptotically stable for the actual nonlinear system.
- If the linearized system is unstable, then the origin is unstable (for the nonlinear system).

3. Lyapunov’s Direct Method

The Lyapunov’s direct method allows to draw conclusions on the stability without using the difficult stability definitions introduced earlier. It gives necessary conditions for the equilibrium to be stable, and is based on the existence of a Lyapunov function.\(^{26}\)

**Lyapunov function:** If, in a ball \( B_{R_0} \), the scalar function \( V(k) \) is positive definite and the function \( V(k+1) - V(k) \) is negative semi-definite, then \( V(k) \) is said to be a Lyapunov function for the system.
With this definition of Lyapunov functions, the main Lyapunov theorem for stability can be introduced.

**Theorem:** If there exists a Lyapunov function, then the origin is stable. If, actually, the function \( V(k+1) - V(k) \) is negative definite in \( B_{R_0} \), then the stability is asymptotic.

This theorem can be applied directly once the update equation (4) is known. The difficulty lies in the process of searching for the Lyapunov function. Its non-existence does not prove the instability of the origin, but the impossibility for this method to prove stability. Other methods have then to be applied.

4. **Graphical Method**

In some cases, the interdependencies of the subsystems of the decentralized system can give interesting properties to the solutions of these problems. One of those cases occur when a small number of disciplines are closely related to one another and almost isolated from the rest of the design. Equation (6) shows an example for 2 interrelated disciplines and with quadratic update equations.

\[
\begin{align*}
x(k+1) &= ax(k) + by^2(k) \\
y(k+1) &= cx(k) + dx^2(k)
\end{align*}
\]

For those coupled equations, an independent update equation can easily be found:

\[
x(k+1) = f(x(k-1))
\]

This equation can be plotted in the \((x(k-1), x(k+1))\) plane, and the stability can be inferred from the position of the curves with respect to the quadrants and to the lines \(x(k+1) = \pm x(k-1)\). This method is used in the case study where more explanation is given.

This section presents the main methods adapted from Nonlinear Control theory and that can be used in solving stability of equilibriums in decentralized design. The next section introduces a case study on which some of these methods can be tested.

V. **Design of a Pressure Vessel**

As a way to illustrate the methods introduced in the previous section, the design of a thin-walled pressure vessel with hemispherical ends, as shown in Figure 2 is used. The nomenclature for this case study, taken from,\(^{17}\) is presented in Table 2.

<table>
<thead>
<tr>
<th>W</th>
<th>Weight of the pressure vessel (lbs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>V</td>
<td>Volume ((in^3))</td>
</tr>
<tr>
<td>R</td>
<td>Radius ((in))</td>
</tr>
<tr>
<td>T</td>
<td>Thickness ((in))</td>
</tr>
<tr>
<td>L</td>
<td>Length ((in))</td>
</tr>
<tr>
<td>P</td>
<td>Pressure inside the cylinder ((klb))</td>
</tr>
<tr>
<td>(S_t)</td>
<td>Material allowable tensile strength ((klb))</td>
</tr>
<tr>
<td>(\rho)</td>
<td>Density of the material ((lbs/in^3))</td>
</tr>
<tr>
<td>(\sigma_{circ})</td>
<td>Circumferential stress ((lbs/in^2))</td>
</tr>
</tbody>
</table>

Table 2. Nomenclature of the Pressure Vessel

The vessel is to withstand a specified internal pressure \(P\) and the material is also specified. There are two objectives: to minimize the weight and to maximize the volume of the cylinder, both subject to stress and geometry constraints. Although this is not naturally a multi-player problem, we consider in this paper that the design involves 2 design teams: 1) player VOL who wishes to maximize the volume and controls \(R\) and \(L\), and 2) player WGT who wishes to minimize the weight of the vessel and controls \(T\). The objectives and constraints of the two players are shown in Tables 3 and 4.
Figure 2. Thin-Walled Pressure Vessel

**PLAYER VOL**

Maximize  
\[ V(R, L) = \frac{4}{3}\pi R^3 + \pi R^2 L \]

Design Variables  
\( R \) and \( L \)

Stress constraint  
\[ \sigma_{\text{circ}} = \frac{PR}{T} \leq S_t \]

Geometric constraints  
\( 5T - R \leq 0 \)
\( R + T - 40 \leq 0 \)
\( L + 2R + 2T - 150 \leq 0 \)

Side constraints  
\( 0.1 \leq R \leq 36 \)
\( 0.1 \leq L \leq 140 \)

**Table 3. Model of Player VOL**

**PLAYER WGT**

Minimize  
\[ W(R, T, L) = \rho \left[ \frac{4}{3}\pi(R+T)^3 + \pi(R+T)^2L - \left( \frac{4}{3}\pi R^3 + \pi R^2 L \right) \right] \]

Design Variables  
\( T \)

Stress constraint  
\[ \sigma_{\text{circ}} = \frac{PR}{T} \leq S_t \]

Geometric constraints  
\( 5T - R \leq 0 \)
\( R + T - 40 \leq 0 \)
\( L + 2R + 2T - 150 \leq 0 \)

Side constraints  
\( 0.5 \leq T \leq 6 \)

**Table 4. Model of Player WGT**

The specific data (problem constants) for this problem is as follows:

\[ P = 3.89 \text{ klb} \]
\[ S_t = 35.0 \text{ klb} \]
\[ \rho = 0.283 \text{ lbs/}_{\text{in}}^3 \]
As discussed in Section III, the equilibriums of the design space are located at the intersection of the players’ RRS, and we use Response Surface Methodology to approximate those RRS. Equations (8) and (9) show the RRS of each player if we choose to approximate them by a quadratic function. \(^\text{17}\)

\[
\begin{align*}
\text{VOL} & \quad R(T) = 29.29 + 14.75T - 10.01T^2 \\
L(T) & = 85.45 - 34.45T + 20.10T^2 \\
\text{WGT} & \quad T(R, L) = 2 + 1.75R + 0.2445R^2 \\
\end{align*}
\]

Equations (8) and (9) show the RRS of each player if we choose to approximate them by a quadratic function.

The intersection of those Rational Reaction Sets gives us the Nash solution(s). There is only one intersection here, therefore only one equilibrium point, given in Equation (10).

\[
\begin{align*}
R^N & = 28.4 \text{ in} \\
L^N & = 87.5 \text{ in} \\
T^N & = 3.09 \text{ in} \\
\end{align*}
\]

In the remaining of the paper, we use normalized values from -1 to 1 for the three design variables. As mentioned in Section IV, in order to study the stability of this equilibrium, we first need to move it back to the origin. This is done by substituting \(x\) by \((x + x_N)\), where \(x = [R, L, T]^T\) is the state vector, and \(x_N\) is the state vector evaluated at the equilibrium. The new update equation is shown in Equation (11).

\[
\begin{align*}
R(k + 1) & = 0.887T(k) - 0.558T(k)^2 \\
L(k + 1) & = -0.525T(k) + 0.287T(k)^2 \\
T(k + 1) & = 0.739R(k) + 0.089R(k)^2 \\
\end{align*}
\]

The first method to investigate the stability in the neighborhood of the equilibrium is linearization. In this case, it is straightforward, as we just need to keep the linear terms of Equation (11), and put it in a matrix format, as shown in Equation (5), to find the state equation \(A\).

\[
A = \begin{bmatrix}
0 & 0 & 0.887 \\
0 & 0 & -0.525 \\
0.739 & 0 & 0
\end{bmatrix}
\]

A simple check of the eigenvalues of \(A\) gives us the stability properties of the linearized system.

\[
\text{Eigenvalues of } A = \{0, 0.8096, -0.8096\}
\]

The stability is determined by the value of the spectral radius (maximum absolute value of the eigenvalues of a matrix), which is here: \(r_\sigma(A) = 0.8096\). Since it is strictly less than one, the linearized system is strictly stable, and therefore the initial nonlinear system is asymptotically stable.

The next method to be tried, in order to extend the notion of stability to a larger region around the equilibrium point, is the Lyapunov’s direct method. For this example, several candidate Lyapunov functions have been tried. All of them were positive definite, but unfortunately, none of them verified \(V(k) - V(k - 1)\) to be negative semi-definite. This does not mean that it is impossible to find a Lyapunov function and that the system is not stable, but rather that other methods have to be tried.

In this case, the graphical method can be used. Indeed, we can notice from Equation (11) that the design variables \(R\) and \(T\) are closely related to each other, while design variable \(L\) is just a function of \(T\). Starting from Equation (14),

\[
\begin{align*}
R(k + 1) & = 0.887T(k) - 0.558T(k)^2 \\
T(k + 1) & = 0.739R(k) + 0.089R(k)^2 \\
\end{align*}
\]
an update equation linking $T(k + 1)$ to $T(k - 1)$ can easily be found and plotted in the $(T(k - 1), T(k + 1))$ plane. This graph is shown in Figure 3, along with the drawing of the quadrants.

Effective insight on the stability can be gained by studying this figure. Indeed the update curve is always in the first and third quadrant, which means that the sign of the design variable does not change from one iteration to the next one. Besides, the curve lies within the lines $T(k + 1) = \pm T(k)$, for every value between -1 and 1. That means that, no matter where the design process is started, the state vector will always end up at the origin, thus proving stability of this equilibrium in the whole design space. The example of an initial condition of $T_0 = 0.8$ is shown in Figure 3, and shows the first iterations of the design process, and the convergence towards the origin of the design space.

Therefore, the domain of attraction of this Nash Equilibrium is the entire design space. Note that it is not necessary that the whole design space is in the domain of attraction, even if there is a unique equilibrium, as some initial conditions might create a divergent pattern as noted in and.

**Conclusion**

Most engineering systems are multidisciplinary in nature and therefore require knowledge from several design teams. This, along with other constraints, forces the decentralization of decisions. Therefore, the decision makers involved in this design process need to understand the mechanism of this process in order to find a final optimal design. This paper goes a step further in understanding the dynamics of these decentralized systems by studying the stability of the equilibrium points of the design space. It extends the existing results about stability to more complex problems with highly nonlinear objectives and constraints. This is done by introducing new approaches, directly adapted from Nonlinear Control Theory. A case study presents the first direct applications of these methods on a real design example.

The future work of this research will concentrate on validating all these methods on another case study. Indeed, we still need to demonstrate the implementation of some of the methods presented theoretically in this paper.
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