

# A Formal Approach to Handling Conflicts in Multiattribute Group Decision Making

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*Supporting the decision of a group in engineering design is a challenging and complicated problem when issues like consensus and compromise must be taken into account. In this paper, we present the foundations of the group hypothetical equivalents and inequivalents method and two fundamental extensions making it applicable to new classes of group decision problems. The first extension focuses on updating the formulation to place unequal importance on the preferences of the group members. The formulation presented in this paper allows team leaders to emphasize the input from certain group members based on experience or other factors. The second extension focuses on the theoretical implications of using a general class of aggregation functions. Illustration and validation of the developments are presented using a vehicle selection problem. Data from ten engineering design groups are used to demonstrate the application of the method.*  
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## 1 Introduction

Typically in decision making, we face the need to make trade-offs. We have to pay more for a faster computer, expect existing problems with a used car, or wait in longer lines for higher airport security. More specifically, in engineering design, there is typically no one alternative that is clearly superior over all other alternatives across every criterion. When the design decision is multicriterion in nature, common challenges include mathematically aggregating diverse and conflicting criteria, quantitatively capturing a decision maker's preferences over multiple criteria, and providing an ordered scale (ordinal or cardinal) of alternatives that can be used to select the best one, making the decision a very complex problem. Therefore, how to make the "best" decision when choosing from among a set of alternatives in a design process has been a common problem in research and application in engineering design.

A number of methods have been proposed to support multiattribute decision making. In [1], a Pareto frontier-based approach to concept selection in conceptual engineering design is presented. This approach, termed the *s*-Pareto frontier-based concept selection, identifies an aggregate Pareto frontier of unique design concepts and subsequent variations on these concepts. The approach is expanded in [2] to handle uncertainty while providing an interactive visualization tool. In [3], graph theory and linear physical programming are used to identify promising combinations of subsystems in complex systems design.

A fuzzy outranking relation is developed to rank design concepts in an imprecise and uncertain design environment [4]. In [5], an integrated approach to product selection incorporating designer's preferences, customers' preferences, market competition and general uncertainty is presented using the basic concepts of multiattribute utility theory. In [6], an implicit value function is used to iteratively narrow down design alternatives from a designer's stated preferences through a series of attribute trade-off trials that effectively eliminate lower-value designs. An approach to modeling preferences accurately and consistently is presented in [7] and

used to generate and select from a set of locally acceptable solutions. As a way to determine and utilize attribute importances in selection decisions, indifference points are used with a general class of aggregation functions in the method of imprecision [8]. However, finding two designs that are of exact equivalent value to a decision maker can be a challenging and time-consuming task [9].

The hypothetical equivalents and inequivalents method (HEIM) [5] expands the concepts of indifference relationships found in [8,10], and allows decision makers to express a preference of one alternative over another in order to effectively determine attribute importances. The method is easy to use, only requiring the elicitation of preferences over a series of hypothetical product design alternatives. HEIM has since been expanded to effectively identify a single, robust solution [11] and to handle alternatives with uncertain attributes [12]. In addition, a general consistency check denoted as the preference consistency check [13] is developed to ensure that HEIM provides single and consistent solution.

Although providing effective decision support in various environments and under certain assumptions, these approaches are applicable to decisions being made by a single decision maker or a design team with a single set of unified preferences. However, in a realistic design team environment, different opinions and conflicting preferences among group members can be expected, and these approaches would generally not be applicable. Therefore, in this paper, we expand and further develop HEIM to support multiattribute group decision making to address some significant challenges in aggregating group member preferences.

One significant issue in group decision making is Arrow's impossibility theorem [14], which has been discussed in the engineering design domain [15–17]. One of the properties of the theorem refers to the notion of consistency in group preferences (i.e., if *A* is preferred to *B* and *B* is preferred to *C*, then *A* is preferred to *C*), which cannot be guaranteed in groups and is typically only possible by employing a dictatorship. Indeed, some methods that ensure consistency in group decision making, such as cardinal utility [18], an additive value function [19], and an additive utility function [20], are effective in aggregating group preferences, but at the expense of the freedom of the individuals in the group. Individual preferences are structured in some way, and the methods tend to become relatively unfriendly to use for the individuals. In addition, product design companies are hesitant to use methods such as these because individuals are required to have some fundamental background in decision theory in order to use and apply

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the methods. Therefore, only those decision tools that are easy to use are adapted by product design companies even though these tools may have limitations or theoretical flaws. For example, the analytical hierarchy process (AHP) method has been proven to have some theoretical flaws [21], and the “house of quality” is shown to use unreliable information [22], but they are still widely used in industry because of their flexibility, ease of use, and ease of implementation. Therefore, when developing new design tools, they have to provide not only effective decision support, but also be user-friendly and easy to implement.

By understanding the problems and needs in multiattribute group decision making, a basic group formulation of HEIM is developed (group hypothetical equivalents and inequivalents) in [23] to aggregate individuals’ preferences. Instead of restricting the freedom of each group member, group-HEIM allows individuals to freely express preferences over a number of hypothetical alternatives (HA). The level of conflict or differences between the aggregated group preferences is then explored in group-HEIM to find a best “compromise” solution. In this paper, we are presenting the theoretical foundations of group-HEIM, demonstrating how it reveals the source of conflict in the group preferences and identifies the most preferred group solution. Group-HEIM is similar to the multiattribute approach described in [19] because it uses stated equality preferences from the decision maker based on hypothetical alternatives. However, group-HEIM is different because it accommodates inequality preference statements and is easily scalable to problems with many attributes because it avoids having to address preferential independence or reduction of dimensionality when there are three or more attributes.

In addition, two important extensions of group-HEIM are developed and presented in this paper. The first focuses on the handling and aggregation of unequal group members. A number of pedagogical studies on group decision making acknowledge that the contributions, experience, and knowledge of every member on a team is rarely equal [24,25]. It would make sense many times to give more importance to a team member who may have more experience, education, or domain-specific knowledge. Therefore, it is necessary to accommodate this natural and appropriate bias in a group’s decision making to benefit the team’s outcomes. In [26], preferences from unequal group members are integrated using relative weights. However, determining exact weights for group members is subject to the same challenges and limitations as determining exact weights for attributes or objectives [27–29]. We present an improved group formulation that implicitly accounts for group member differences in the decision formulation itself and investigate how the solution is affected in their favor.

The second extension focuses on investigating the effect of different aggregation function formulations on the resulting decision. In previous work, the group-HEIM formulation relied on a weighted-sum method ( $L_1$ -Norm) to determine the value of each alternative. Although commonly used and potentially sound in discrete choice problems, weighted-sum methods generally run the risk of missing “optimal” options because they rely exclusively on importance weights [27]. Therefore, a complete aggregation function requires an additional parameter to specify the level of compensation between attributes in addition to the importance weights [30]. In this paper, the effect of using this type of aggregation function, as proposed in [31], is studied and the relationship between the level of conflict, identified directly by group-HEIM, and the level of the compensation parameter is illustrated, both empirically and theoretically. In Sec. 2, the foundation of group-HEIM and the developments are put into the context of the entire method.

## 2 Group-HEIM: A Formulation for Group Decision Making

In this section, we present the foundations of the formulation of group-HEIM with emphasis on the new developments, which are the focus of this paper. First, an overview of the basic mechanics of both group-HEIM and HEIM is presented.

**2.1 Basic Premise.** In HEIM for individual decision makers and in group-HEIM for groups, the decision maker(s) do not have to specify precise attribute weights individually or as a group, easing the burden of the decision process and eliminating a typically challenging task in multiattribute decision making. The attribute weights are found through setting up and comparing a set of hypothetical alternatives (HA). The “equivalents” part of the method allows decision makers to make statements such as “hypothetical alternatives  $A^1$  and  $A^2$  are equivalent in value to me.” On the other hand, the “inequivalents” part of the method allows decision makers to make statements such as “I prefer hypothetical alternative  $A^1$  over  $A^2$ .” Therefore, when a preference is stated, by either equivalence or inequivalence, a constraint is formulated.

The equality constraints are developed based on the stated preference of “I prefer alternatives  $A^1$  and  $A^2$  equally.” In other words, the values of these alternatives are equal, resulting in

$$V(A^1) = V(A^2) \quad \text{or} \quad V(A^1) - V(A^2) = 0 \quad (1)$$

The value of an alternative (alternative  $A^j$  in this case) is given by

$$V(A^j) = \left[ \sum_{i=1}^n (w_i r_i^j)^s \right]^{\frac{1}{s}} \quad (2)$$

Equation (2) is a simplified version of aggregation function based on the Method of Imprecision [32] as the sum of weights is always equal to one.  $r_i^j$  is the normalized rating of alternative  $A^j$  on attribute  $i$ . For instance, for a set of vehicle alternatives whose attributes include miles per gallon (MPG), the MPG rating for one of the vehicles would simply be the vehicle’s MPG value, normalized between 0 and 1 using the highest and lowest MPG values of all the candidate vehicles. The parameter  $s$  can be interpreted as a measure of compensation, or trade-off. Higher values of  $s$  indicate a greater willingness to allow high preference for one criterion to compensate for lower values of another. As reported in [31], the aggregation function of Eq. (2) satisfies a set of axioms that an aggregation function, appropriate for rational design decision making, must obey. Note that when  $s=1$ , Eq. (2) becomes a  $L_1$ -Norm (weighted-sum) and when  $s=2$ , Eq. (2) becomes a  $L_2$ -Norm [33]. Although the previous implementations of HEIM have used the  $L_1$ -Norm for simplicity, other forms of value functions with different parameters are investigated in later sections of this paper.

The inequality constraints are developed based on the stated preference of “I prefer  $A^1$  over  $A^2$ .” In other words, the value of alternative  $A^1$  is more than alternative  $A^2$ , as shown in the following:

$$V(A^1) > V(A^2)$$

$$V(A^1) - V(A^2) > 0$$

$$V(A^1) - V(A^2) + \delta \geq 0 \quad (3)$$

where  $\delta$  is a small positive number to ensure inequality. In Sec. 2.2, the steps of group-HEIM are detailed, including the fundamental formulations, to illustrate how these preference statements are used to find the attribute weights for multiattribute group decisions.

**2.2 Group-HEIM Outline.** There are six steps in group-HEIM to process and aggregate group preferences. A vehicle example is used to demonstrate how group-HEIM assists a team of five design engineers to identify and specify attributes.

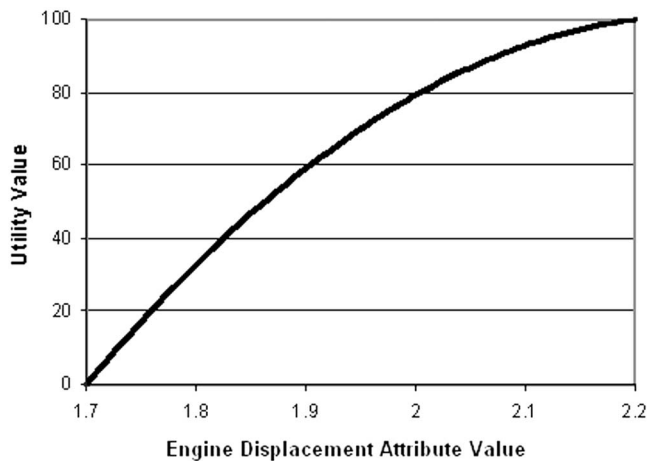
**Table 1 Attributes and ranges**

Attribute	Lower limit	Upper limit
Engine displacement	1.7	2.2
Horsepower (hp)	127	145
Maximum mile per gallon (MPG)	31	40
Price (\$)	11,995	13,884
Acceleration (0–60 mph)	7.9	10.5

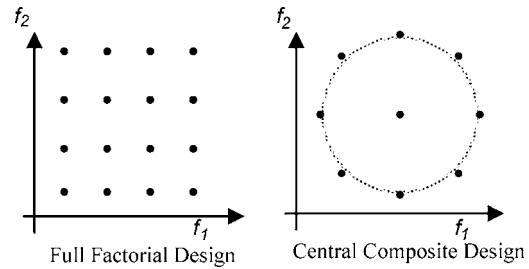
*Step 1. Identify the Important Attributes and Attribute Range.* Group-HEIM is not able to identify the absence of an important attribute. If an unimportant attribute is included in the process, group-HEIM will indicate the attribute’s limited role with a low weighting factor. Some techniques, such as conjoint analysis, factor analysis and value-focused thinking [34–36], can be used to identify the key attributes. The range of each attribute can be identified based on the manufacturing capability, constraint requirements, or production cost. The information of attribute and attribute range is then used to identify the strength of preference (SOP) of each individual, which is the next step. Table 1 shows an example of five attributes and their ranges for the new compact car design.

*Step 2. Determine the Strength of Preference (SOP) for Each Individual on each Attribute.* These strength of preferences (other notations in the literature include single attribute utility functions) reflect the decision makers’ true preferences on a certain attribute for a given attribute range. The objective is to capture decision makers’ risk attitude toward different attributes. Figure 1 shows a “risk averse” preference of one of the designers for an engine displacement design over a given range. A nonlinear SOP representation is suggested to better reflect the decision maker’s true preferences. Since the attribute ranges have been predefined from the previous step, it is assumed that the designers’ utility for the most preferred value in the range can be set to unity (saturated). This not only facilitates calculations but also is quite reasonable since the basic selection is discrete in nature and the maximum utility for the most preferred attribute value across all possible alternatives could certainly be considered to be unity.

However, if there is a change in any of the attribute ranges (e.g., a new alternative is introduced), it is necessary to renormalize the utility values (with monotonicity assumed) and recapture the new SOP. It is noted that the conflict within a single attribute could potentially be identified using the curvature of the SOP (i.e., identifying a risk-averse member and a risk-prone member). However, in multiattribute decision problems, conflicts arise from trade-offs among the attributes. Therefore, although two members could have different opinions about a single attribute, they might



**Fig. 1 Preference for engine displacement design over a given range**



**Fig. 2 Examples of sampling the performance space**

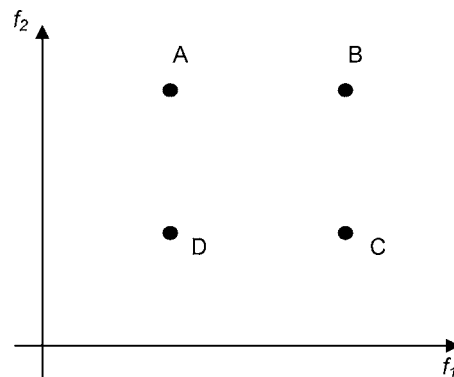
have the same preference regarding the relative importance of the attribute. Some ways to assess these SOPs are lottery, midlevel splitting, and indifferent point methods [37–39]. Note that each team member identifies their own SOPs for each attribute since the SOP information is used to set up the individual hypothetical alternatives next.

*Step 3. Set up Hypothetical Alternatives and Elicit Preference Structure for Each Group Member.* In this step, the information used to construct the equality and inequality constraints is elicited from each group member. Hypothetical alternatives (HA) are a key ingredient in group-HEIM. It is critical that when evaluating two hypothetical alternatives, one is not dominated by another. That is, one alternative should not be better across all attributes compared to another alternative. There should be some balance of attributes that forces the decision maker to process and make a trade-off when choosing one of the hypothetical alternatives.

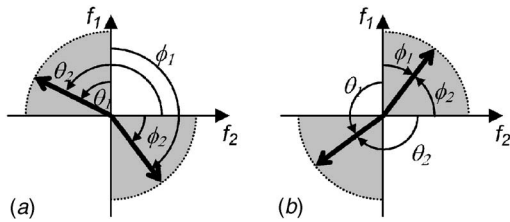
In order to develop an appropriate set of HAs, the attribute space is sampled in a structured and balanced manner, using design of experiments (DOE) [40]. In most applications of DOE, the input factors are design or control variables. However, in this application of DOE, the input factors are the product attributes relevant to the decision. The attributes dictate the performance of the product and do not reflect a specific design configuration, size, material, or any other typical design variable. They only reflect the performance of a hypothetical design alternative. To be more specific, DOE customizes the hypothetical design alternatives for each decision maker based on the strength of preferences (risk attitude) provided by the decision makers in step 2. By developing a different set of HAs for each team member, it allows for equal treatment across preference strengths, which helps when minimizing conflict among group members

In Fig. 2, two different DOE examples used to sample the performance space are shown. The attributes,  $f_1$  and  $f_2$ , are two conflicting attributes in a typical 2D performance space, where the objective is to maximize both attributes (e.g., miles per gallon and horsepower). The example on the left of Fig. 2 is a full factorial design while a central composite design is shown on the right.

Once the HAs are set up (sampled) appropriately in the perfor-



**Fig. 3 Typical 2D performance space**



**Fig. 4 Possible alternative comparisons: (a) useful projection, (b) non-useful projection**

mance space, the decision makers are then asked to compare pairs of HAs in order to set up the preference constraints for the optimization formulation. The main purpose of the comparison is to identify the most preferred location in the performance space so that the relative attribute weights can then be solved for. However, some comparisons may not lend any useful or nonredundant information. Figure 3 is presented to further explain the types of HA comparisons that can be made.

In Fig. 3, assume A, B, C, and D are four sampled hypothetical alternatives from the performance space with two conflicting attributes,  $f_1$  and  $f_2$ , that are to be maximized. Therefore, we need to compare the alternatives that will provide useful information. If we compare D to B, no useful information will be obtained since B dominates D by having higher values for both  $f_1$  and  $f_2$ . Alternatively, if  $f_1$  and  $f_2$  are to be minimized, then D will dominate B. However, when comparing C and A, the decision maker has to decide which attribute to sacrifice in order to get a better value on the other attribute. This comparison, as a result, provides useful constraint information in the optimization formulation that is used to solve for attribute importances in step 5.

In Fig. 4, the useful and nonuseful projections of comparison, in vector form and normalized to the origin, are shown along with the angles with respect to the coordinate axes. Assuming the attributes are all being minimized or all being maximized, Fig. 4(a) shows the useful vectors of comparison of A to C (fourth quadrant) or C to A (second quadrant). Fig. 4(b) shows the projections in the first and third quadrant where no useful information will be generated, since one alternative along one of these directions will always dominate the other alternative. From the simple two-dimensional (2D) illustration in Fig. 4, we can conclude that in order to provide a useful comparison, the angles between the projection and the coordinate axes,  $(\theta_1, \theta_2)$  or  $(\phi_1, \phi_2)$ , cannot all be between either (0 deg, 90 deg) or (90 deg, 180 deg). As long as one of the angles is between (0 deg, 90 deg) and the other is between (90 deg, 180 deg), a useful comparison can be made, as shown in Fig. 4(a).

This analysis can be expanded to the general case of  $n$  attributes. Assume two design points in  $n$ -dimensional space, A and B, are represented in coordinate vector form as  $\bar{A}=[a_1 \ a_2 \ \dots \ a_n]^T$  and  $\bar{B}=[b_1 \ b_2 \ \dots \ b_n]^T$ . The projection from point A to point B can be found as  $\bar{P}=[p_1 \ p_2 \ \dots \ p_n]^T$ , where  $\bar{P}=\bar{A}-\bar{B}$ . Then, the angles between the projection and all the coordinate axes can be found using a form of the standard vector dot product formulation, as shown in

$$\begin{bmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{bmatrix} = \cos^{-1} \left[ \left( \frac{1}{\sqrt{p_1^2 + p_2^2 + \dots + p_n^2}} \right) \right] \cdot \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_n \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{bmatrix} \quad (4)$$

where the identity matrix represents the coordinate axes in  $n$ -dimensional space. The following conditions determine whether or not the projection is useful:

**Table 2 D-optimal experimental design and corresponding attribute values**

No. (alt)	Factor 1 (engine)	Factor 2 (Hp)	Factor 3 (MPG)	Factor 4 (price)	Factor 5 (acceler.)
1 (A)	2 (1.85)	2 (135)	2 (34)	2 (12,500)	1 (10.5)
2 (B)	3 (2.2)	2 (135)	3 (38)	1 (11,995)	1 (10.5)
3 (C)	2 (1.85)	3 (145)	1 (31)	3 (13,884)	2 (9.5)
4 (D)	2 (1.85)	1 (127)	3 (38)	1 (11,995)	3 (7.9)
5 (E)	3 (2.2)	2 (135)	2 (34)	3 (13,884)	3 (7.9)
6 (F)	3 (2.2)	1 (127)	1 (31)	2 (12,500)	1 (10.5)
7 (G)	3 (2.2)	3 (145)	3 (38)	2 (12,500)	3 (7.9)
8 (H)	1 (1.7)	2 (135)	3 (38)	2 (12,500)	2 (9.5)
9 (I)	1 (1.7)	1 (127)	3 (38)	3 (13,884)	1 (10.5)
10 (J)	1 (1.7)	3 (145)	2 (34)	1 (11,995)	1 (10.5)
11 (K)	3 (2.2)	1 (127)	2 (34)	1 (11,995)	2 (9.5)
12 (L)	1 (1.7)	2 (135)	1 (31)	1 (11,995)	3 (7.9)

- Nonuseful condition: If  $0 \text{ deg} \leq \theta_i \leq 90 \text{ deg} \ \forall \theta_i, i = 1, \dots, n$  or  $90 \text{ deg} \leq \theta_i \leq 180 \text{ deg} \ \forall \theta_i, i = 1, \dots, n$ , then the projection does not provide any useful information.
- Useful condition: If  $\exists$  at least one  $\theta_i \ni 0 \text{ deg} \leq \theta_i \leq 90 \text{ deg}$  and  $\exists$  at least one  $\theta_i \ni 90 \text{ deg} \leq \theta_i \leq 180 \text{ deg}$ , then the projection does provide useful comparison.

To improve the efficiency of preference elicitation, the total number of hypothetical alternatives should be kept minimal while maximizing the amount of useful information. In this work, we use a D-optimal experimental design to both effectively sample the performance space while minimizing the number of hypothetical alternatives and maximizing the value of the information gained from each alternative comparison. For example, a five-factor, three-level D-optimal design matrix is used in the vehicle selection problem (Table 2) since there are five attributes each with three possible levels. The levels 1, 2, and 3 in Table 2 correspond to the utility values of 0, 50, and 100, respectively, from the SOP formulations in step 2. Table 2 also shows the corresponding numerical values (in parentheses) for each attribute using the strength of preferences for one of the designers. For instance, the utility value of 50 for the designer is a 1.85 L engine based on the strength of preference indicated in step 2.

Note that although Alt. G dominates Alt. A., the hypothetical alternatives can be partitioned into smaller groups for useful comparisons. Partitioning the alternatives not only allows designers to make useful comparisons, but also eases the mental burden for a designer by only having to consider small number of alternatives at a time. The trade-offs expressed by designers can then be used to identify their preferences for the relative importance of attributes.

*Step 4. Aggregate Individuals' Preference Structures as an Optimization Problem.* The stated preferences (both equivalent and inequivalent) between HAs provided by each decision maker are aggregated into a single optimization problem formulation. In HEIM (for a single decision maker), Eqs. (1) and (3) are used as the constraints. However, in group-HEIM, if the constraints from each group member preference in step 3 were placed together into one optimization problem, there would most likely be no feasible solution. This is because it is not likely that all members of the group would agree on all their assessments. For example, if one group member preferred alternative  $A^1$  over alternative  $A^2$  and another preferred  $A^2$  over alternative  $A^1$ , this would result in two opposite constraints of the form given in Eq. (3), preventing a feasible solution from being found.

Consistency among a group cannot be guaranteed [16], and in practice, it is rare for every member of a product development group to have exactly the same preferences as well. Some common group decision methods have avoided inconsistency issues in aggregated group preferences by limiting the freedom of individuals in the group [15–17,41,42]. Group-HEIM instead allows individuals to freely express preferences over a number of HAs and

**Table 3 Hypothetical alternatives preference structures for five individuals**

Designer	Preference structures		
#1	$C > B > D \phi A$	$E > H > F \phi G$	$L > J > K > I$
#2	$A > B > D > C$	$F > G > E > H$	$L > J > K > I$
#3	$C > B > A > D$	$H > F > G > E$	$J > K > I > L$
#4	$D > B > C > A$	$H > F > G > E$	$J > L > K > I$
#5	$C > D > B > A$	$H > F > G > E$	$J > L > K > I$

then explores the level of conflict or inconsistency in the aggregated group preferences. Group-HEIM acknowledges that consistency cannot be guaranteed and identifies and minimizes the inconsistency in the group.

Based on the least-distance approximation method [43], group-HEIM extends the single decision-maker formulation in HEIM by adding variables into the constraints in Eqs. (1) and (3). These variables, called *compromise variables*, are used to identify the conflicts in preferences among group members. The basic formulation is shown in the following:

Minimize

$$\sum_{\{\phi\}} (x_{jk})^p + \sum_{\{\sim\}} |z_{uw}|^p$$

subject to

$$V(A^j) - V(A^k) + x_{jk} \geq 0$$

for all inequality preferences

$$V(A^u) - V(A^v) + z_{uw} = 0$$

for all equality preferences

$$\sum w_i = 1$$

Side constraints

$$w_i \geq 0, \quad x_{jk} \geq 0 \quad (5)$$

where  $i$  is the number of attributes,  $p$  is an integer,  $r_i^j$  is the rating of alternative  $A^j$  on attribute  $i$ ,  $\{\phi\}$  is the set of inequality preferences,  $\{\sim\}$  is the set of equality preferences,  $x_{jk}$  is the compromise variable for the inequality preference of alternatives  $A^j$  and  $A^k$ ,  $z_{uw}$  is the compromise variable for the equality preference of alternatives  $A^u$  and  $A^v$ , and  $V(A^i)$  is the value of alternative  $A^i$ . These compromise variables are both calculated and minimized, since they appear in the objective function in Eq. (5).

In general terms, the objective is to minimize the level of conflict by treating each group member equally. However, this paper also provides a modified formulation to emphasize certain group members over others in Sec. 2.3. Equation (5) provides a unique and single solution even when conflict occurs in the preference structures. If all the preferences are consistent and can all be satisfied, all the compromise variables will be equal to zero. If a set of preferences is conflicting, the corresponding compromise variables in Eq. (5) will be nonzero.

Table 3 shows the preference structures for the five designers on the hypothetical alternatives from Table 2, where  $>$  indicates "preferred to." Again, note that the HAs are different for each individual because the HAs are generated based on the individual's strength of preferences. From the preference structures, 32 preference pairs are identified as shown in Table 4. The preference pairs are used to formulate the inequality constraints in the optimization formulation in Eq. (5). Note that there are no equality

**Table 4 32 preference pairs**

$A > B$	$C > A$	$E > F$	$G > E$	$J > K$
$A > D$	$C > D$	$E > G$	$G > H$	$J > I$
$A > C$	$C > B$	$E > H$	$H > F$	$K > I$
$B > D$	$D > A$	$F > E$	$H > G$	$K > L$
$B > A$	$D > B$	$F > G$	$H > E$	$L > I$
$B > C$	$D > C$	$F > H$	$I > L$	$L > J$
			$J > L$	$L > K$

constraints in these preference structures.

By using the formulation given in Eqs. (5) and (2) with  $s=1$  in the value function, the constraints for preference  $A > B$  and  $A > D$ , for example, can be written as,

$$G_{AB} = -0.5w_1 - 0.5w_3 + x_{AB} \geq 0$$

$$G_{AD} = 0.5w_2 + 0.5w_3 + 0.5w_4 - 0.5w_5 + x_{AD} \geq 0 \quad (6)$$

The remaining 30 constraints can be written in a similar format. The purpose of the objective function in Eq. (5) is to minimize the compromise variables, or the amount of conflict in the group's preferences. The compromise variables in the least-distance approximation,  $x_{jk}$  and  $z_{uw}$ , are utilized to ensure that equality and inequality preferences are satisfied. For instance, in the case of a conflicting preference, the compromise variable will be nonzero to ensure the inequality preference is always greater than or equal to zero. Also observe that the objective function in Eq. (5) is a convex space; thus, the constraint summing the weights to 1 is needed to avoid trivial solutions.

*Step 5. Solve for the Attribute Weights.* Depending on the value of  $p$  shown in Eq. (5), various optimization methods can be applied to solve the formulation. For example, if  $p=1$ , then the formulation is a linear problem and can be solved using linear programming. In this work, we use  $p=2$ , and the optimization solver in EXCEL (based on the GRG method). For the vehicle problem, the solution is unique and the set of weights is found to be  $\bar{w} = [0.26, 0.23, 0.26, 0.17, 0.09]^T$ . The nonzero compromised variables identify the conflicting objectives as well. For instance, the compromised variable for  $x_{AB}$  is 0.174, indicating that the preference of  $A \phi B$  cannot be satisfied. Indeed, if we study the preference structures in Table 3, there are four group members that prefer  $B \phi A$ , whereas only one group member prefers  $A > B$ , supporting the fact that  $A > B$  should not be satisfied.

*Step 6. Make Selection Decision using Attribute Weights.* After the attribute weights are identified and specified, the alternative with the highest value (Eq. (2)) can be identified. At this point, group-HEIM has identified the conflicting preferences or inconsistency among the group members, and the group has the option to go back and focus on these preferences and attempt to get consensus on them, as presented in [23]. In addition, the group can decide if the solution should be shifted in favor of certain group members because of their valuable experience and reliable insights. This can be done directly in group-HEIM as illustrated in Sec. 2.3.

**2.3 Unequal Group Members.** A group leader can decide to discount or elevate the opinion of certain members of the group based on their experience, their education, or their contribution to the team thus far. Being able to place more emphasis or importance on certain group members' opinions is not only necessary when some group members may have valuable experience and reliable insight on a certain product line, but may also may help speed up a design process and improve the probability of success of a product [44]. We accomplish the emphasis of certain group members directly in the optimization formulation of Eq. (5) by adding constraints that limit the compromise of a certain group member to be less than another. For instance, if a group leader determines that designer  $n$  is more important than designer  $m$

**Table 5 Attribute data for vehicle alternatives**

Alternative	Attributes and relative weights				
	$w_1$	$w_2$	$w_3$	$w_4$	$w_5$
	Engine (L)	Horsepower (hp)	MPG	Price (\$)	Acceleration (0–60 mph)
Car 1	2.0	145	36	12,906	8.6
Car 2	1.7	127	38	13,470	10.5
Car 3	2.2	140	33	11,995	9.9
Car 4	1.8	130	40	13,065	9.5
Car 5	2.0	132	36	12,917	10.0
Car 6	2.0	130	31	13,315	10.4
Car 7	2.2	140	33	13,884	7.9
Car 8	2.0	135	33	12,781	9.8

(e.g., by using the number of years of experience with the company, or any other reasonable metric), the formulation is updated as shown in Eq. (7). In Eq. (7), the additional constraint that the sum of all compromise variables for designer  $n$ ,  $\sum x_{jk|n}$ , is less than the sum of all compromise variables for designer  $m$ ,  $\sum x_{jk|m}$ , which implies that designer  $n$  is more important than designer  $m$ . In other words, the solution should compromise the preferences of designer  $n$  less than those of designer  $m$ . Note that the only difference between Eqs. (7) and (5) is the added constraint for all unequal group members. Explanation of the other terms in Eq. (7) is given in Eq. (5).

Minimize

$$\sum_{\{\phi\}} (x_{jk})^p + \sum_{\{\sim\}} |z_{uv}|^p$$

subject to

$$V(A^j) - V(A^k) + x_{jk} \geq 0$$

for all inequality preferences

$$V(A^u) - V(A^v) + z_{uv} = 0$$

for all equality preferences

$$\sum x_{jk|m} - \sum x_{jk|n} \geq \delta$$

for all unequal group members

$$\sum w_i = 1$$

Side constraints

$$w_i \geq 0, \quad x_{jk} \geq 0 \tag{7}$$

With the given understanding of the basics of the group-HEIM process, including the new developments emphasized in this paper, in Sec. 3 we exercise the approach. We focus on studying the results from the two primary extensions of this paper: being able to weight the preferences of each group member differently and investigating the effects of different aggregation functions.

**Table 6 Preference structures for the group members**

Designer	Preference structures			
1	$C > B > A$	$E > D > F$	$I > H > G$	$K > L > J$
2	$C > A > B$	$E > D > F$	$H > I > G$	$J > L > K$
3	$C > B > A$	$E > D > F$	$G > I > H$	$L > K > J$
4	$C > B > A$	$E > F > D$	$H > I > G$	$J > K > L$

**Table 7 Hypothetical alternatives for designer 1, group 1**

HA	Engine (L)	Horsepower (hp)	MPG	Price (\$)	Acc. (0–60 mph)
A	2	137.8	34.6	13,128	10.5
B	2.2	137.8	40	13,884	10.5
C	2.2	145	40	13,128	7.9
D	2	127	40	13,884	7.9
E	2.2	137.8	34.6	11,995	7.9
F	2.2	127	31	13,128	10.5
G	2	145	31	11,995	8.9
H	1.7	137.8	40	13,128	8.9
I	1.7	127	40	11,995	10.5
J	1.7	145	34.6	13,884	10.5
K	2.2	127	34.6	13,884	8.9
L	1.7	137.8	31	13,884	7.9

### 3 Exercising the Approach

The case study is based on the vehicle product specification used in Sec. 2. Here, we exercise the approach using ten actual design groups consisting of four to six mechanical and aerospace senior engineering students each. Each group is given the same attribute information in Table 1, and they follow the steps described in Sec. 2.2 to identify a set of attribute weights for the group. An automated MS EXCEL interface is used by the individuals in the groups to determine their strength of preference functions using lottery questions. The EXCEL interface then creates hypothetical alternatives based on the responses, elicits the individual preferences over the HAs and solves for the attribute weights using the EXCEL SOLVER. With the attribute weights, each group then evaluates the overall value of the eight concept cars in Table 5. We then use the group responses to study the effect of imposing unequal importance to the group member preferences and the effect of different value aggregation functions. This problem is simplistic and not realistic in terms of how car manufacturers select vehicle designs. It is rather meant to illustrate the contributions of this work.

**3.1 Relative Importance of Group Members.** To demonstrate the application of Eq. (7), only group 1 is used to illustrate the approach. There were four members of group 1, and Table 6 shows the preference structures for each group member across the sets of hypothetical alternatives. There were four sets of three hypothetical alternatives generated using a D-optimal design. The set of hypothetical alternatives for designer 1 are shown in Table 7. The other group members do not necessarily have the same hypothetical alternatives, since their strength of preferences assessments (step 2) are most likely different from designer 1.

Each group member has eight compromise variables corresponding to the eight unique constraints created by the preferences in Table 6. These are shown in Table 8. We can use these variables to study the effect of making the group member importance unequal.

One of the main assumptions in this work is that the preferences for all the group members, *individually*, are consistent (i.e., transitivity is satisfied for each individual). For example, if designer 1 prefers C to B and B to A, then they prefer C to A. This is illustrated by examining the following three constraints:

$$G_{CB} = 0.5w_2 + 0.5w_4 + w_5 + x_{CB} \geq 0$$

**Table 8 Compromise variables for each group member**

Designer	Compromised variables							
1	$x_{CB}$	$x_{BA}$	$x_{ED}$	$x_{DF}$	$x_{IH}$	$x_{HG}$	$x_{KL}$	$x_{LJ}$
2	$x_{CA}$	$x_{AB}$	$x_{ED}$	$x_{DF}$	$x_{HI}$	$x_{IG}$	$x_{JL}$	$x_{LK}$
3	$x_{CB}$	$x_{BA}$	$x_{ED}$	$x_{DF}$	$x_{GI}$	$x_{IH}$	$x_{LK}$	$x_{KJ}$
4	$x_{CB}$	$x_{BA}$	$x_{EF}$	$x_{FD}$	$x_{HI}$	$x_{IG}$	$x_{JK}$	$x_{KL}$

**Table 9 Comparison of compromise variables**

Designer	Sum of compromise values	
	Original solution	Solution w/unequal members
1	0.1835	0.1578
2	0.1435	0.1579
3	0.1009	0.1580
4	0.2640	0.2758

$$G_{BA} = 0.5w_1 + 0.5w_3 - 0.5w_4 + x_{BA} \geq 0$$

$$G_{CA} = 0.5w_1 + 0.5w_2 + 0.5w_3 + w_5 + x_{CA} \geq 0$$

Note that the constraint  $G_{CA}(C>A)$  is the sum of the constraints  $G_{CB}(C>B)$  and  $G_{BA}(B>A)$ , when  $x_{CA}=(x_{CB}+x_{BA})$  and is therefore redundant in the formulation.

Assuming individual consistency, now assume that the opinion of designer 1 is the most important, the opinion of designer 2 is the second most important, the opinion of designer 3 is the third most important, and the opinion of designer 4 is the least important. In no way are we promoting treating people unequally, but are simply acknowledging that it may be beneficial to consider the technical insight of certain team members more than others.

The additional constraints necessary to model these importance are shown in Eq. (8) where  $\delta=0.001$  to ensure inequality. The first constraint implies that the sum of the compromise variables (as shown in Table 8) for designer 1 must be *less than* the sum of the compromised variables for designer 2, indicating that the preferences for designer 1 should be adhered to more than the preferences of designer 2. The basic group formulation is updated with these constraints and solved again (using Eq. (2) with  $s=1$ ).

$$\begin{aligned} \sum x_{jk}|_{\text{Designer 2}} - \sum x_{jk}|_{\text{Designer 1}} &\geq \delta \\ \sum x_{jk}|_{\text{Designer 3}} - \sum x_{jk}|_{\text{Designer 2}} &\geq \delta \\ \sum x_{jk}|_{\text{Designer 4}} - \sum x_{jk}|_{\text{Designer 3}} &\geq \delta \end{aligned} \quad (8)$$

The original set of weights with equal group members (i.e., without using Eq. (8)) are as follows:

$$\bar{w} = [0.083, 0.147, 0.228, 0.403, 0.138]^T$$

The chosen vehicle for the group is vehicle 3. With the integration of the relative importance constraints from Eq. (8), the new attribute weights are as follows:

$$\bar{w} = [0.090, 0.136, 0.232, 0.398, 0.144]^T$$

The chosen vehicle for the group is again 3. The same vehicle was found with the two different sets of weights for reasons that are discussed in Sec. 3.2. However, what can be noted is that the solution has shifted in favor of designer 1. Table 9 shows the sum of the compromise variables for each group member. The results illustrate that the sum of the compromise variables for designer 1 decrease from the original formulation. In addition, designer 1 now has the smallest amount of compromise among all the designers, followed by designer 2, designer 3, and lastly, designer 4. This matches the intent of the constraints formulated in Eq. (8). By placing a priority on the information from designer 1, Table 9 also clearly shows that the other three designers must compromise their preferences more than the original formulation.

Another form of validation is to study how the set of attribute weights changes to better accommodate the preferences of designer 1. The weight for horsepower (the second weight) decreases the most indicating that designer 1 may not really place too much emphasis on horsepower. Investigating the hypothetical alternatives in Table 7 and the preference structure in Table 6, the

**Table 10 Relative attribute weights and selected vehicle:  $L_1$ -Norm**

Group	Attribute weights					Car No.	Value
	$w_1$	$w_2$	$w_3$	$w_4$	$w_5$		
1	0.083	0.147	0.228	<b>0.403</b>	0.138	3	0.6752
2	0.157	0.178	0.299	<b>0.331</b>	0.036	3	0.6905
3	0.046	0.146	0.268	<b>0.337</b>	0.204	3	0.5940
4	0.156	0.187	<b>0.308</b>	0.258	0.091	3	0.6385
5	0.083	0.147	0.228	<b>0.403</b>	0.138	3	0.6752
6	0.083	0.147	0.228	<b>0.403</b>	0.138	3	0.6752
7	0.101	0.166	0.277	<b>0.299</b>	0.157	3	0.6175
8	0.036	0.157	<b>0.319</b>	0.238	0.251	1	0.5962
9	0.083	0.147	0.228	<b>0.403</b>	0.138	3	0.6752
10	0.237	0.164	<b>0.326</b>	0.225	0.048	3	0.6640

most preferred alternatives in the third and fourth sets (I and K, respectively) both have the lowest horsepower available, clearly indicating its lack of importance to designer 1. On the other hand, the weight for engine size (the first weight) increases the most, indicating that designer 1 seems to emphasize the engine size in their choices. Indeed, three of the four most preferred alternatives (C, E, and K) have the largest engine size of their set. Similar explanations can be offered for the other changes in attribute weights as well. The results shown thus far are based on an  $L_1$ -Norm aggregation function ( $s=1$ ), or weighted sum. Since weighted-sum methods run the risk of missing optimal options as described previously, it is necessary to study the effect of other aggregation functions which may identify a better solution.

**3.2 Value Function Study.** In the previous implementations of HEIM and group-HEIM, the simplest aggregation was used ( $s=1$  in Eq. (2)). However, in this paper we investigate the impact of the value function with different  $s$  values, including the  $L_2$ -norm shown in Eq. (9) [33],

$$V(A^j) = \sqrt{\sum_{i=1}^n (w_i r_i^j)^2} \quad (9)$$

By applying value aggregation functions with different values of  $s$  into the optimization formulation in Eq. (5), different sets of attribute weights are expected, as well as different sums of compromise variables.

**3.2.1 Value Function With  $s=1(L_1$ -Norm).** The resulting weights for the ten design groups, their chosen vehicle, and the overall value of the winning vehicle are shown in Table 10 using Eq. (2) with  $s=1(L_1$ -Norm). Note that the  $L_1$ -Norm representation is used to find the value in the final column, and the values in the group-HEIM formulation (Eq. (5)) in order to solve for the attribute weights.

Table 10 shows that the most preferred car for nine of the ten groups is vehicle 3. The attribute weights in bold for each group show the most important attribute. Nine groups indicated vehicle price as the most important attribute. Four groups have the exact same relative attribute weights, which indicates that the group members have the same or very close to the same preference structures between the groups.

For the nine groups that chose vehicle 3 in Table 10, there were six different sets of weights. Four of these six sets had price as the most important attribute, while two had miles per gallon as the most important attribute. This consistency in the final selection, even with differences in the attribute weights and importance can be explained by a simple two-dimensional representation shown in Fig. 5. The black circles represent actual nondominated vehicles, and their aggregation is represented by the thick line, approximating the nondominated set of vehicles (this is hypothetical and only meant to explain the results from Table 2).

It is well known that using an  $L_1$ -norm in two dimensions is

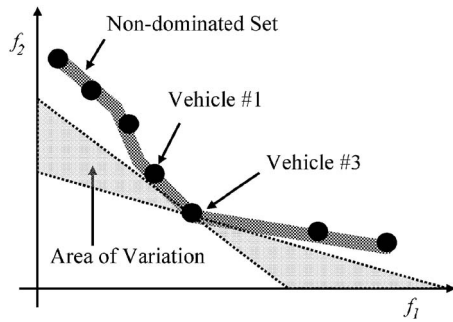


Fig. 5 Variations in weights

equivalent to projecting a line with a given slope (determined by a ratio of the attribute weights) toward the nondominated set until a solution from the set is contacted. This is depicted in this figure for two different lines, representing a range of possible attribute weights. As long as the weights are within this range, the most preferred vehicle will be identified as vehicle 3, as shown in Fig. 5. Also evident is vehicle 1, which is close to vehicle 3. If the weights were to change slightly, then vehicle 1 may be identified as the most preferred alternative. Therefore, the groups can have different sets of weights and even different most important attributes, but yet still identify the same preferred alternative.

**3.2.2 Value Function With  $s=2(L_2\text{-Norm})$ .** To compare to the results from Table 10, the results when Eq. (2) with  $s=2(L_2\text{-Norm})$  is used are shown in Table 11. The  $L_2$ -norm is a common approach to finding nondominated solutions in nonconvex spaces. The exact same preference structures for all group members were used. The values in Eq. (5) and the last column in Table 11 are calculated using Eq. (9). The attribute weight values for each group have changed, as would be expected, since the constraints in Eq. (5) were altered slightly with the new value function. Eight groups now indicate that price is the most important attribute. The most preferred vehicle has stayed the same for every group except for group 3, which switched from vehicle 3 to 1.

Investigating this a bit further, in Table 12, the difference in values for vehicles 3 and 1 are shown for group 3 using both norms. When the  $L_1$ -norm additive model is used, the value difference between the first and second ranked car is 5.3%. However, when the  $L_2$ -norm is used, the difference is only 0.02%. Therefore, by comparing the differences in the values, vehicle 3 could still be considered as the most preferred car for group 3 since the difference in the values is so small (Table 12). This observation, nevertheless, suggests that perhaps the top two or three alternatives should be kept under consideration in the design process until more information can clearly distinguish them.

As demonstrated here, the  $L_1$ - and  $L_2$ -Norms inherently use a

Table 11 Relative attribute weights and selected vehicle:  $L_2$ -Norm

Group	Attribute weights					Car No.	Value
	$w_1$	$w_2$	$w_3$	$w_4$	$w_5$		
1	0.178	0.191	0.199	0.249	<b>0.184</b>	3	0.6513
2	0.194	0.194	0.220	0.235	<b>0.157</b>	3	0.6538
3	0.141	0.189	0.229	0.233	<b>0.208</b>	1	0.6093
4	0.189	0.192	0.227	<b>0.217</b>	0.176	3	0.6353
5	0.178	0.191	0.199	0.249	<b>0.184</b>	3	0.6513
6	0.178	0.191	0.199	0.249	<b>0.184</b>	3	0.6513
7	0.183	0.199	0.214	<b>0.213</b>	0.191	3	0.6315
8	0.127	0.184	0.245	<b>0.235</b>	0.209	1	0.6063
9	0.178	0.191	0.199	0.249	<b>0.184</b>	3	0.6513
10	0.205	0.191	0.232	<b>0.210</b>	0.161	3	0.6421

Table 12 Decision of group 3 using different value functions

Rank	$L_1$ -Norm model		$L_2$ -Norm method	
	Vehicle No.	Value	Vehicle No.	Value
First	3	0.5840	1	0.6093
Second	1	0.5528	3	0.6092
	Difference	5.3%	Difference	0.02%

predetermined  $s$  value for the value function in Eq. (2). This artificial constraint is imposed on the groups so that they make the same level of compensation among attributes regardless of the number of conflicting preferences each group has. However, a weighted sum, or any other predetermined aggregation procedure, is overly and inappropriately constraining [8]. Therefore, the level of compensation for each group should be situation dependent, as the "one-size-fits-all" value function can lead to incorrect results. To determine the most appropriate choice of compensation and attribute weights, "indifference points" [8] among a nondominated set are used to effectively identify a set of attribute weights and the most appropriate compensation (value of  $s$ ). However, finding two designs that are equivalent value to a decision maker can be a challenging and time-consuming task [9]. Therefore, in Sec. 3.2.3, we demonstrate how group-HEIM uses hypothetical inequivalents or "difference points" to solve for the attribute weights and the appropriate level of compensation necessary by making  $s$  a variable in Eq. (5).

**3.2.3 Value Function With  $s$  unrestricted.** To use the aggregation function and the concept of hypothetical inequivalence to find the best combination of attribute weights and compensation,  $s$  becomes a design variable in the optimization formulation of Eq. (5). It is argued in [45] that values of  $s \leq 0$  are more appropriate for design because they satisfy the annihilation axiom, which says that if the preference for any one attribute of an alternative is zero, then the value of the entire alternative is zero. However, in our studies with over 100 engineering students, engineers, and engineering managers, very few, if any, of them attribute a value of zero to an alternative that has the poorest performance on one attribute. Therefore, values of  $s \leq 0$  that satisfy the annihilation axiom do not seem appropriate for implementations of HEIM and group-HEIM, but current studies are examining the theoretical foundations of this observation.

Table 13 shows the resulting weights for the ten design groups, their chosen vehicles, the overall value of the winning vehicle, and the level of compensation  $s$ .

The choice of  $s$  is not arbitrary; they are determined by the optimization formulation of Eq. (5). The results from Table 13 suggest an inherent relationship between the level of conflict (sum of  $x_{jk}$ ) and the level of compensation (value of  $s$ ). This relationship is shown in Fig. 6, which clearly indicates that the higher the level of conflict, the higher the level of compensation necessary. In other words, the more conflict a group has, the more trade-off is needed for the group to reach a rational and compromised solution. In fact, this proposition can be proved mathematically using Eq. (5) and the properties of the value function.

**PROPOSITION.** Given two hypothetical nondominated alternatives,  $A^j = (\alpha_1, \alpha_2)$  and  $A^k = (\beta_1, \beta_2)$  and assuming without loss of generality that  $\alpha_1 > \beta_1, \beta_2 > \alpha_2$  and that a designer prefers  $A^k$  over  $A^j$ , then

$$\frac{ds}{dx_{jk}} > 0$$

This proposition states that as the level of conflict in a group increases, the level of compensation required to reach a compromised solution using Eq. (5) increases. Note that  $s$  and  $x_{jk}$  are independent variables and this proposition mathematically describes the monotonic behavior between  $s$  and  $x_{jk}$  as shown in Fig.

**Table 13 Results of using compensating aggregation function**

Group	Attribute weights					Car No.	Value	s Value	Sum of $x_{jk}$
	$w_1$	$w_2$	$w_3$	$w_4$	$w_5$				
1	0.077	0.146	0.236	0.407	0.133	3	0.6552	0.8376	0.0315
2	0.173	0.158	0.240	0.317	0.112	3	0.8486	3.7836	0.4071
3	0.046	0.146	0.269	0.333	0.206	3	0.5968	1.0407	0.0000
4	0.199	0.199	0.239	0.192	0.170	3	0.7800	3.0400	0.1689
5	0.077	0.146	0.236	0.407	0.133	3	0.6552	0.8376	0.0315
6	0.077	0.146	0.236	0.407	0.133	3	0.6552	0.8376	0.0315
7	0.144	0.184	0.241	0.261	0.170	3	0.6986	1.7005	0.1248
8	0.046	0.164	0.308	0.240	0.241	3	0.6152	1.3298	0.0338
9	0.077	0.146	0.236	0.407	0.133	3	0.6552	0.8376	0.0315
10	0.212	0.187	0.259	0.207	0.135	3	0.7671	2.5380	0.1633

6.

*Proof.* To prove this proposition, we must first understand the properties of the aggregation function. As mentioned previously, the values of  $s \leq 0$  that satisfy the annihilation axiom do not seem appropriate for implementations of HEIM and group-HEIM. Therefore, the maximum operator from [30] is used and the value functions for  $A^j$  and  $A^k$  are

$$V(A^j) = (w_1\alpha_1^s + w_2\alpha_2^s)^{1/s} \quad V(A^k) = (w_1\beta_1^s + w_2\beta_2^s)^{1/s} \quad (10)$$

From [30],  $V(A^j)$  and  $V(A^k)$  are nondecreasing as a function of  $s$ , and if a larger value of  $s$  is used, then the overall preference for all nondominated points will be larger.

To prove this proposition, we can use a simplified version of the optimization statement of Eq. (5),

Minimize

$$x_{jk} + x_{kj}$$

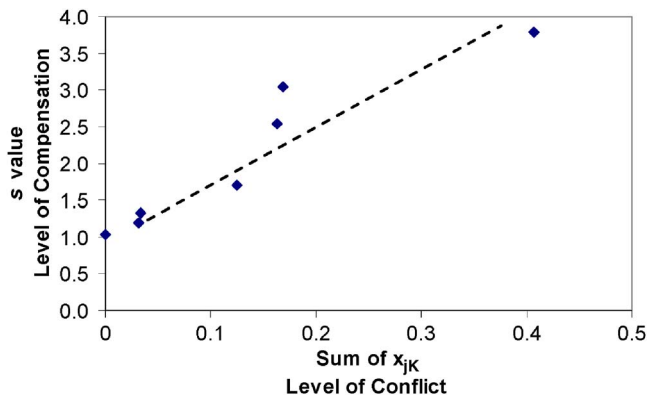
subject to

$$V(A^j) - V(A^k) + x_{jk} \geq 0$$

$$V(A^k) - V(A^j) + x_{kj} \geq 0$$

$$x_{jk}, x_{kj} \geq 0 \quad (11)$$

Recall that  $\alpha_1 > \beta_1, \beta_2 > \alpha_2$ , which implies that  $A^k$  is more compensating than  $A^j$  (i.e.,  $A^k$  has a higher performance on one attribute that makes up for lower performance on other attributes). Assuming that a designer prefers  $A^k$  over  $A^j$ , then the preference  $V(A^k) > V(A^j)$  is satisfied while  $V(A^j) > V(A^k)$  is not. Therefore, the first constraint in Eq. (11) represents a conflict giving  $V(A^j) - V(A^k) < 0$ , and  $x_{jk} > 0$ ; the second constraint in Eq. (11) is satisfied



**Fig. 6 Relationship between the level of conflict and level of compensation**

and becomes  $V(A^k) - V(A^j) > 0$ , with  $x_{kj} = 0$ . The optimization statement can be further simplified as

Minimize

$$x_{jk}$$

subject to

$$V(A^j) - V(A^k) + x_{jk} = 0$$

$$V(A^k) - V(A^j) > 0 \quad (12)$$

From the assumptions (i.e.,  $A^k$  is more compensating than  $A^j$ ), differentiating the second constraint of Eq. (12) with respect to  $s$  gives

$$\frac{d}{ds}[V(A^k) - V(A^j)] > 0 \quad (13)$$

Equation (13) illustrates that the difference of  $V(A^k) - V(A^j)$  not only depends on  $s$ , but increases as  $s$  increases. The first constraint in Eq. (12) gives  $x_{jk} = V(A^k) - V(A^j)$ , and since  $x_{jk}$  is to be minimized, the difference between  $V(A^k)$  and  $V(A^j)$  is minimized as well. Therefore, the value of  $s$  is driven by the minimization of  $x_{jk}$ .

In order to qualify this relationship, Eq. (13) can be rewritten as an equality equation as follows:

$$\frac{d}{ds}[V(A^k) - V(A^j)] = c$$

$$d[V(A^k) - V(A^j)] = cds$$

$$d[V(A^j) - V(A^k)] = -cds \quad (14)$$

where  $c$  is a positive constant.

Furthermore, the first constraint from Eq. (5) can be differentiated with respect to  $s$  giving us

$$\frac{d}{ds}[V(A^j) - V(A^k)] + \frac{dx_{jk}}{ds} = 0$$

$$\frac{dx_{jk}}{ds} = \left\{ \frac{d}{ds}[V(A^j) - V(A^k)] \right\}$$

$$dx_{jk} = -d[V(A^j) - V(A^k)]$$

Replacing  $d[V(A^j) - V(A^k)]$  with Eq. (14),

$$dx_{jk} = cds$$

$$\frac{ds}{dx_{jk}} = \frac{1}{c} \quad (15)$$

Since  $c$  is a positive constant, we can conclude that,  $ds/dx_{jk} > 0$ ,

**Table 14 Nondominated alternatives**

Alternative	Attribute 1	Attribute 2
A	0.1	0.9
B	0.3	0.6
C	0.75	0.4
D	0.9	0.1

which proves the proposition stated previously.

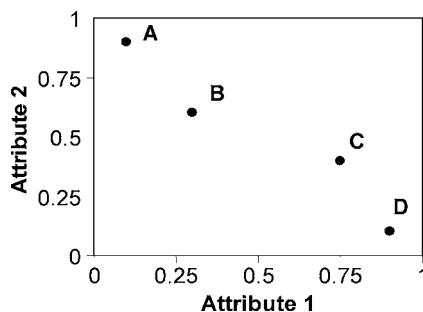
This demonstrates how group-HEIM establishes a relationship between the level of conflict and the level of compensation necessary to make rational engineering selection decisions using the concept of hypothetical inequivalence. The group can subsequently understand how much trade-off they have made in order to reach a compromised solution.

Although positive values of  $s$  are found in these studies, there are some limitations on large values of  $s$ . It is shown that higher values of  $s$  cannot reach all nondominated (Pareto) points on nonconvex frontiers in [45]. However, the objective of group-HEIM is not to populate a Pareto frontier but rather to identify the most favorable alternative on the frontier. Yet with higher values of  $s$ , there may be some limitations on what alternatives can be identified with group-HEIM. In Table 14, a simple example is shown that has four nondominated alternatives and their attribute values. Figure 7 illustrates the location of these alternatives in the performance space. Note that alternative B is in the nonconvex region of the discontinuous nondominated frontier (assume maximization of the attributes).

A study was first conducted that varied the values of the attribute weights in Eq. (2) with  $s=1$  and  $s=2$  (i.e.,  $L_1$ - and  $L_2$ -norms). No combination of weights in group-HEIM allowed for alternative B to be identified as the best alternative. However, when  $s$  was allowed to vary (along with the weights), all four of the alternatives were able to be identified. Table 15 shows the values of the attribute weights and  $s$  necessary for each alternative to be identified as the highest valued alternative. As is clearly seen, with weights of (0.30, 0.70) and  $s=0.001$ , alternative B is identified as the winner. Although simple in nature, this illustrates that it is possible for group-HEIM to identify alternatives on both the convex and nonconvex portions of the nondominated frontier using the general aggregation function of Eq. (2). This is primarily because in the current results (Table 13) and other studies, the values of  $s$  are never very large, but are typically between 0 and 4, allowing for all nondominated alternatives to be identified. A number of general observations and conclusions are made in the next section.

#### 4 Observations and Conclusions

Although group-HEIM was introduced in [23], this paper has established some of the fundamental formulations in group-HEIM and has extended and improved group-HEIM in two important areas. First, it develops the basic foundation for using value ag-

**Fig. 7 Nondominated alternatives****Table 15 Resulting weights and  $s$  values for each winning alternatives**

Alternative	$w_1$	$w_2$	$s$ value	Alternative value
A	0.00	1.00	1.000	0.900
B	0.30	0.70	0.001	0.487
C	0.53	0.47	0.200	0.564
D	1.00	0.00	1.000	0.900

gregation approaches other than the common  $L_1$ -norm. Second, it establishes an effective formulation for being able to emphasize the opinions of certain group members.

Although the group-HEIM formulation in Eq. (5) includes constraints for indifference relationships, finding “indifference points,” can be a challenging and time-consuming task as noted in [9], specifically in the context of constructing utility functions. Therefore, the indifference relationship is not used specifically in this paper, but has been used in other applications. Although the new constraints in Eq. (7) set up a lexicographic priority structure among group members, additional constraints could be used to numerically specify the relative importance of group members (e.g., a constraint on the ratio of  $\sum x_{jk}$  among group members). The approach avoids having to assign weights to group members, which would create a number of implementation and solution challenges.

These important developments to group-HEIM help establish its foundations as an easy-to-use, sound approach to supporting decision making when groups of engineers or managers are involved. In related work, a software program is being developed to automate many of the steps in group-HEIM. As mentioned previously, the ten experimental groups completed the decision process using an automated MS EXCEL interface. A significant advantage is that all the preferences of the group members are captured without having to meet collectively. Therefore, when the group does meet, they can devote their time to discussing the conflicts that have been identified and build consensus. The decision maker’s primary responsibility as an individual is to create accurate strength of preference functions (single-attribute utility functions) for each attribute and then to state their preferences over pairs of hypothetical alternatives. Beyond this, everything else can be automatically processed using the underlying mathematical foundation of group-HEIM.

Current work includes investigating the consistency of each group member [13] and to allow for inconsistent individual preferences to be identified and corrected quickly. Group consistency is rare and cannot be guaranteed, but individual consistency should be established before applying group-HEIM or any other decision support tool. If decision makers are inconsistent with their preferences, then the validity and accuracy of the resulting decision cannot be trusted. Also, current work is focused on studying the appropriate number of constraints necessary in the group formulation. Too many constraints may result in a solution that is largely unusable, while too few constraints may result in an underconstrained problem with no meaningful solution.

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