Robust product family consolidation and selection

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Robust product family consolidation and selection

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The design and development of effective product lines is a challenge in modern industry. Companies must balance diverse product families that satisfy wide-ranging customer demands with practical business needs such as integrative manufacturing processes and material and supplier selection. In a global marketplace, this is an increasingly difficult challenge. In this paper, the issue of consolidating an existing product family is addressed. Specifically, the hypothetical equivalents and inequivalents method (HEIM) is utilised in order to select an optimal product family configuration. In previous uses, HEIM has been shown to assist a decision maker in selecting design concepts when performance attributes conflict and trade-offs must be made. In the extension of HEIM presented in this work, the constraints of an optimisation problem are formulated using two different value functions, and common solutions are identified in order to select an optimal family of staplers. The result is then compared with the result found using a multi-attribute utility theory (MAUT)-based approach. While each method has its advantages and disadvantages, and MAUT provides a necessary first step for product family consolidation and selection, a robust solution is achieved through HEIM.

Keywords: product families; multiobjective optimisation; knowledge representation; robust concept selection; multiattribute decision-making

1. Introduction

Today’s highly demanding and globally competitive marketplace drives companies into managing the pressure by developing product platforms and designing families of products. Gone are the days when industry designed and produced individual products without some consciousness of component commonality and product diversification. Although companies certainly face risks in developing product families, such as weak common platforms, over-design of low-end variants and longer initial development times (Van Vuuren and Halman 2001), rewards exist as well. If the family is successful, individual products generally require less development time (Gupta and Souder 1998). Additional benefits include reduced system complexity and decreased local and off-shore manufacturing costs (Simpson 2004).

There are two recognised approaches to product family design (Simpson \textit{et al.} 2001). The first, a top–down (proactive platform) approach, exists when a company strategically manages...
and develops a family of products based on a product platform and its derivatives (Simpson et al. 2001). The product platform concept exploration method (PPCEM) (Simpson 1998) and the variation-based platform design methodology (VBPDM) (Nayak et al. 2000) are examples in product family design literature which offer means to design families based on platforms. In industry, many design strategies are developed using this type of approach. For instance, AeroAstro Inc. developed a multipurpose radio platform with 24 derivative products to meet a wide variety of spacecraft radio applications (Caffrey et al. 2002). The company did so by identifying eight common themes to be present in each radio. AeroAstro Inc. then carefully studied a radio market segmentation grid for growth opportunities.

In another case, Apple Computer Inc. developed the iPod product line based on a similar single modular design platform, keeping firmware, iTunes software and control architecture the same while scaling internal components, LCDs and housings (Marion and Simpson 2006). This platform approach has allowed Apple to target different market segments while providing customers a set of branded experiences and content-owners with value capture solutions (Sawhney et al. 2006). Other companies routinely practise such scaling of components. A leader in the aerospace industry, Boeing, developed the 777 airliner based on a scalable platform that allowed both the flight range and aircraft capacity to be varied (Sabbagh 1996). Today, the Boeing 777 is available in six models, five of which span multiple market segments in passenger travel and one is used for cargo shipping (Boeing 2008). Comparable instances of a top–down approach to product family design are found throughout all types of industries (Yamanouchi 1989, Rothwell and Gardiner 1990, Paula 1997).

The second general approach to product family design, a bottom–up (reactive design) approach, exists when a company redesigns or consolidates a group of distinct products to standardise components and improve economies of scale (Simpson et al. 2001). Many companies utilise this type of approach for product family design. In the software industry, for example, companies often use a reactive approach for platform design because it is a logical tool for managing the evolution of the software (Riva and Del Rosso 2002). In Meyer and Seliger (1998), it is noted that reactive approaches are often used in conjunction with top–down approaches, as companies look both to create new platforms and derive platforms from existing software. This type of reactive platform derivation and consolidation is seen in other industries as well. In the 1970s, after years of developing different tools and motors, Black & Decker redesigned and standardised their product line to increase commonality, thereby reducing costs (Meyer and Lehnerd 1997). This not only increased profitability among existing products but also helped spawn new products which were based on the newly developed standardised components (Meyer and Lehnerd 1997).

There exist many other bottom–up methods that companies employ for product family design. One of them involves consolidating individual products in an existing family to obtain an optimal family architecture, and recently multi-attribute utility theory (MAUT) has been utilised in this context to consolidate a family of staplers (Thevenot et al. 2006). This approach involves a decision maker who chooses from among a number of alternatives on the basis of two or more alternative attributes. Attributes are first assigned a risk tolerance by finding attribute-specific certainty equivalents. These risk tolerances are then used for the calculation of attribute-specific utility functions. Then, an aggregated utility function is determined by performing trade-off analysis (Keeney and Raiffa 1993). Although Thevenot et al. provide an effective approach for product family consolidation based on MAUT, the focus was not on studying robustness in terms of the optimal product family.

The remainder of this paper presents an approach for selecting an optimal product family that focuses on studying the robustness of the solution with respect to problem formulation and attribute preferences. In Section 2, we introduce the hypothetical equivalents and inequivalents method (HEIM) as an effective bottom–up approach for product family consolidation that overcomes some limitations in previous approaches. In Section 3, we apply HEIM to consolidate a family of
staplers, and in Section 4 we use two different value function formulations to investigate issues of robustness across a wide range of attribute weights determined using HEIM. Also, in Section 4 we compare optimal product family configurations determined using MAUT and HEIM, and we discuss limitations to be considered when using both methods. Finally, in Section 5 we provide some concluding remarks and summarise key areas for future work.

2. Hypothetical equivalents and inequivalents method

The HEIM is an approach to multi-attribute decision-making (See and Lewis 2002, See and Lewis 2004). Its application to product family consolidation and selection, tasks frequently encountered in a dynamic global marketplace, is illustrated in Section 3. In the present section, a general overview of HEIM is provided, starting with discussion of the fundamental optimisation problem formulation of HEIM in Section 2.1. In Section 2.2, we introduce two value functions that are used to generate constraints for the optimisation problem and conclude with recent applications and extensions of HEIM.

2.1. General overview

HEIM makes use of a set of hypothetical design alternatives to assess a decision maker’s preferences and, ultimately, rank a set of actual design alternatives. HEIM is similar to conjoint analysis in the fact that it captures the inherent trade-offs that decision makers make when evaluating several attributes together. However, by using HEIM, a set of accurate and robust attribute importances can be calculated by converting preferences into constraints and solving a resulting optimisation problem. The HEIM optimisation problem stems from a decision maker’s stated preferences concerning the equivalence or inequivalence in value of two hypothetical alternatives. In other words, by identifying hypothetical alternatives to be equivalent or inequivalent, a decision maker performs an evaluation based on attribute importance. Attribute weights are then directly calculated using an optimisation problem based on these preferences.

Previous work has used indifference relationships similar to HEIM to determine attribute levels in order to solve for attribute weights (Scott and Antonsson 2000). HEIM diverges from this work in the sense that attribute levels are predetermined for each hypothetical alternative. HEIM requires evaluation of equivalence or inequivalence only after attribute levels are established.

HEIM uses an optimisation formulation to determine attribute weights. Weights are then used to evaluate design alternatives. The generalised optimisation problem formulation of HEIM is shown in the following equation:

\[
\begin{align*}
\text{Minimise} : & \quad \sum_{h(\vec{v})} |z_{uv}| + \sum_{g(\vec{v})} (x_{jk}), \\
\text{subject to :} & \quad h(\vec{w}) = 0, \\
& \quad g(\vec{w}) \leq 0, \\
& \quad \sum_{i=1}^{n} w_i = 1, \\
& \quad w_i \geq 0, \quad x_{jk} \geq 0.001,
\end{align*}
\]

(1)

where \( \vec{w} \) is the vector of attribute weights, \( n \) is the number of attributes and \( w_i \) is the weight of each attribute \( i \). Variables \( z_{uv} \) and \( x_{jk} \) are slack variables that are used in the equality and
inequality constraint formulations \( h(\overline{w}) \) and \( g(\overline{w}) \), respectively, as presented in the next section. The slack variables facilitate finding values for attribute weights that are feasible with respect to the preference constraints. The objective function aims to minimise the sum of the slack variables.

2.2. Constraint formulations

In this paper, we first make use of the traditional \( L_1 \)-norm, which is essentially a weighted sum, to formulate constraints for the optimisation problem. Next, we expand our study beyond the \( L_1 \)-norm to define constraints in terms of the \( L_2 \)-norm, which is commonly used to find non-dominated solutions in non-convex spaces (Das and Dennis 1997, Chen et al. 1999, Miettinen 1999, See and Lewis 2005).

Constraints are established from a given preference structure based on hypothetical alternatives. Equality constraints relate two hypothetical alternatives that have equivalent value to a decision maker. This equivalence is depicted in Equation (2) for hypothetical alternatives \( A^u \) and \( A^v \):

\[
V(A^u) = V(A^v).
\]  

(2)

Here, \( V(A^u) \) represents the value of a hypothetical alternative \( A^u \) and is initially defined using the \( L_1 \)-norm in Equation (3),

\[
V(A^u) \bigg|_{L_1} = \sum_{i=1}^{n} w_i a^u_i.
\]  

(3)

where \( a^u_i \) is the rating of hypothetical alternative \( A^u \) on attribute \( i \). Using the \( L_2 \)-norm, the value of hypothetical alternative \( A^v \) is shown in Equation (4),

\[
V(A^u) \bigg|_{L_2} = \sqrt{\sum_{i=1}^{n} (w_i a^u_i)^2}.
\]  

(4)

Accordingly, for both value function formulations, equality constraints in Equation (1) may be written in the form shown in Equation (5), where the \( z_{uv} \) slack variables are unrestricted in sign:

\[
h(\overline{w}) : V(A^u) - V(A^v) + z_{uv} = 0.
\]  

(5)

Inequality constraints relate two hypothetical alternatives with unequal values to a decision maker. This inequivalence is shown in Equation (6) for hypothetical alternatives \( A^k \) and \( A^j \). Equation (7) presents the form for inequality constraints used in Equation (1) for either the \( L_1 \)-norm or the \( L_2 \)-norm. Note that the lower limit on the \( x_{jk} \) slack variables is set at 0.001, as shown in the general formulation of Equation (1), in order to ensure strict inequality of the preferences.

\[
V(A^k) > V(A^j),
\]  

(6)

\[
g(\overline{w}) : V(A^j) - V(A^k) + x_{jk} \leq 0.
\]  

(7)

Once the constraints of Equation (1) are formulated, there exist a variety of methods for obtaining a solution to the optimisation problem. Previous work has used sequential linear programming (SLP) and a generalised reduced gradient (GRG) approach (See and Lewis 2004, See and Lewis 2006).

In many cases, unless the HEIM optimisation problem is sufficiently constrained, the solution is non-unique. Work has been done by Gurnani et al. (2003) to identify a single preferred solution across attribute weights by strategically formulating additional constraints. Additional HEIM
extensions include an approach to ensure consistent preferences (Kulok and Lewis 2007), an ability to handle uncertainty in attribute satisfaction (Gurnani and Lewis 2005) and a group decision-making formulation (See and Lewis 2004), including an approach to account for unequal group member preferences (See and Lewis 2006).

The next section presents the key extension of HEIM for product family consolidation and selection using a stapler family consolidation example. In Section 4, we investigate different value functions and their influences on feasible attribute weights and demonstrate how one optimal and robust product family can be chosen across a range of weights, reflecting practical design scenarios where designers rarely agree on one specific set of weights.

3. Stapler family consolidation and selection using HEIM

To demonstrate the development of HEIM and its application to product family consolidation and selection, we use an existing family of staplers adapted from Thevenot et al. (2006). Figure 1 depicts the entire approach used in this work, including the associated section where the step is discussed. In addition, Figure 1 demonstrates how the HEIM-based approach presented in this work and the MAUT-based approach presented in Thevenot et al. (2006) are similar and how they diverge in implementation.

3.1. Problem identification

The first step of HEIM when specifically applied to product family consolidation and selection requires an adequate identification of the decision problem. The case study involves a small company currently producing three different types of staplers: models 1, 2 and 3, depicted in Table 1. The company is interested in introducing a fourth stapler, model 4 from Table 1, as a strategic way to improve business and better supply the new revenue opportunities in the global marketplace.

Figure 1. HEIM for product family consolidation flowchart.
Table 1. The stapler family (Thevenot et al. 2006).

<table>
<thead>
<tr>
<th>Model</th>
<th>1 (500)</th>
<th>2 (1000)</th>
<th>3 (2000)</th>
<th>4 (3000)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capacity</td>
<td>2–15 Sheets</td>
<td>2–20 sheets</td>
<td>2–60 sheets</td>
<td>2–100 sheets</td>
</tr>
</tbody>
</table>

The consolidation problem stems from the fact that the company would like to optimise this product family configuration by selecting an optimal subset of staplers from the four models. In the next section, we identify the relevant attributes used in the consolidation selection problem.

3.2. Attribute identification

The next step of HEIM involves the selection of key attributes used to measure an optimal product family. In this case study, attributes are chosen to reflect the trade-offs between design, production and marketing at the product and product family levels. The following three key attributes are assumed to satisfy this requirement:

1. product line commonality index (PCI) (Kota et al. 2000);
2. profit over five years;
3. percent market coverage.

From these attributes, we note that the primary goals of the company in this consolidation process include: (1) decreasing product proliferation, which would also decrease the bill of materials (BOM), (2) increasing short-term profits, and (3) maintaining sufficient market coverage. Thevenot et al. (2006) noted that only a limited set of attributes are used for ease in understanding the method of application and details. Other sets of product attributes could have certainly been used, including a performance-related attribute (e.g. number of stapled pages, lifetime of stapler, force required to staple). However, in order to better compare the method to previous work, these attributes are chosen and are used to assist the company in positioning itself for long-term success.

Attribute data for each family of staplers from Thevenot et al. (2006) is given in Table 2. We note that PCI is a component commonality measure for an entire family discussed in Thevenot and Simpson (2006), profit over five years is a numerical measure for a family’s short-term desirability in the market and percent market coverage measures the family’s ability to reach different groups of consumers.

Table 2. Attribute data for stapler families (Thevenot et al. 2006).

<table>
<thead>
<tr>
<th>Product family</th>
<th>PCI</th>
<th>Profit ($)</th>
<th>Market coverage (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 2 3 4</td>
<td>36.5</td>
<td>45,543,018</td>
<td>80.0</td>
</tr>
<tr>
<td>1 2 3</td>
<td>40.7</td>
<td>36,280,518</td>
<td>70.0</td>
</tr>
<tr>
<td>1 2 4</td>
<td>40.7</td>
<td>26,389,514</td>
<td>60.0</td>
</tr>
<tr>
<td>2 3 4</td>
<td>43.1</td>
<td>48,793,824</td>
<td>80.0</td>
</tr>
<tr>
<td>1 3 4</td>
<td>43.1</td>
<td>39,817,768</td>
<td>80.0</td>
</tr>
<tr>
<td>1 2</td>
<td>59.2</td>
<td>17,127,014</td>
<td>50.0</td>
</tr>
<tr>
<td>1 3</td>
<td>42.9</td>
<td>30,555,268</td>
<td>70.0</td>
</tr>
<tr>
<td>1 4</td>
<td>42.9</td>
<td>20,664,264</td>
<td>60.0</td>
</tr>
<tr>
<td>2 3</td>
<td>42.9</td>
<td>39,531,324</td>
<td>70.0</td>
</tr>
<tr>
<td>2 4</td>
<td>42.9</td>
<td>29,640,320</td>
<td>60.0</td>
</tr>
<tr>
<td>3 4</td>
<td>63.3</td>
<td>28,416,004</td>
<td>30.0</td>
</tr>
</tbody>
</table>
3.3. Normalising the attributes: utility theory-based approach

The next step of HEIM is to account for differing dimensions and units among the attributes. Since using a linear preference scale may not accurately reflect company preferences with respect to each attribute, we use the nonlinear utility functions developed for the stapler family case study in Thevenot et al. (2006), which are based on assumed company preferences.

Figure 2 shows the utility function for PCI. We see that the largest and smallest attribute values for PCI assume utilities of 1.0 and 0.0, respectively. Additionally, it is seen that the utility function depicted in Figure 2 is concave and, hence, associated with a risk-averse attitude, where gains on the smaller end of PCI are preferred to equivalent gains on the larger end.

Similarly, Figures 3 and 4 show utility functions for profit and market coverage. Both of these utility functions are associated with risk-prone attitudes, where gains on the larger end are preferred to equivalent gains on the smaller end.

Table 3 gives utility values for each stapler family. Ultimately, it is with these normalised attribute values that stapler families are ranked.

Figure 2. PCI utility (Thevenot et al. 2006).

Figure 3. Profit utility (Thevenot et al. 2006).

Figure 4. Market coverage utility (Thevenot et al. 2006).
Table 3. Utility values for stapler families (Thevenot et al. 2006).

<table>
<thead>
<tr>
<th>Product family</th>
<th>PCI</th>
<th>Profit</th>
<th>Market coverage</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 2 3 4</td>
<td>0.000</td>
<td>0.721</td>
<td>1.000</td>
</tr>
<tr>
<td>1 2 3</td>
<td>0.195</td>
<td>0.269</td>
<td>0.590</td>
</tr>
<tr>
<td>1 2 4</td>
<td>0.195</td>
<td>0.073</td>
<td>0.332</td>
</tr>
<tr>
<td>2 3 4</td>
<td>0.303</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>1 3 4</td>
<td>0.303</td>
<td>0.397</td>
<td>1.000</td>
</tr>
<tr>
<td>1 2</td>
<td>0.882</td>
<td>0.000</td>
<td>0.168</td>
</tr>
<tr>
<td>1 3</td>
<td>0.291</td>
<td>0.134</td>
<td>0.590</td>
</tr>
<tr>
<td>1 4</td>
<td>0.291</td>
<td>0.073</td>
<td>0.332</td>
</tr>
<tr>
<td>2 3</td>
<td>0.291</td>
<td>0.385</td>
<td>0.590</td>
</tr>
<tr>
<td>2 4</td>
<td>0.291</td>
<td>0.119</td>
<td>0.332</td>
</tr>
<tr>
<td>3 4</td>
<td>1.000</td>
<td>0.100</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Table 4. Fractional factorial experimental design.

<table>
<thead>
<tr>
<th>Design (hypothetical family)</th>
<th>Factor 1 (PCI, w₁)</th>
<th>Factor 2 (Profit, w₂)</th>
<th>Factor 3 (Market coverage, w₃)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1 (0.0)</td>
<td>1 (0.0)</td>
<td>1 (0.0)</td>
</tr>
<tr>
<td>B</td>
<td>2 (0.5)</td>
<td>2 (0.5)</td>
<td>3 (1.0)</td>
</tr>
<tr>
<td>C</td>
<td>3 (1.0)</td>
<td>3 (1.0)</td>
<td>2 (0.5)</td>
</tr>
<tr>
<td>D</td>
<td>1 (0.0)</td>
<td>2 (0.5)</td>
<td>2 (0.5)</td>
</tr>
<tr>
<td>E</td>
<td>2 (0.5)</td>
<td>3 (1.0)</td>
<td>1 (0.0)</td>
</tr>
<tr>
<td>F</td>
<td>3 (1.0)</td>
<td>1 (0.0)</td>
<td>3 (1.0)</td>
</tr>
<tr>
<td>G</td>
<td>1 (0.0)</td>
<td>3 (1.0)</td>
<td>3 (1.0)</td>
</tr>
<tr>
<td>H</td>
<td>2 (0.5)</td>
<td>1 (0.0)</td>
<td>2 (0.5)</td>
</tr>
<tr>
<td>I</td>
<td>3 (1.0)</td>
<td>2 (0.5)</td>
<td>1 (0.0)</td>
</tr>
</tbody>
</table>

The next step in Thevenot et al. (2006) evaluates attribute trade-offs and aggregates alternative utilities. However, the HEIM procedure instead assesses decision-maker preferences among hypothetical product families in order to determine relative attribute importances, as shown in the next section.

3.4. Generating hypothetical product families

The next step of HEIM is the creation of hypothetical product families that are used in assessing a decision maker’s preferences. The process of doing so requires a sufficient sampling of the attribute space. In a problem with three attributes, we use a fractional factorial experimental design with three factors and three levels (Atkinson and Doney 1992), as shown in Table 4. Levels 1–3 correspond to attribute utility values of 0.0, 0.5 and 1.0, respectively, also shown parenthetically in Table 4. Thus, this experiment design generates nine hypothetical product families (A–I).

We note that hypothetical family A is a dominated alternative, and we discuss this issue further in the next section. The utility values in Table 4 correspond to actual attribute values shown in Table 5. These hypothetical product family attribute values are then used to develop the preference structure of the decision maker in the next section.

3.5. Developing a preference structure

The next step of HEIM is the development of a preference structure based on the hypothetical family attribute data in Table 5. In this paper, hypothetical families are divided into three groups
Table 5. Attribute data for hypothetical families.

<table>
<thead>
<tr>
<th>Hypothetical family</th>
<th>PCI</th>
<th>Profit ($)</th>
<th>Market coverage (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>36.5</td>
<td>17,127,014</td>
<td>30.0</td>
</tr>
<tr>
<td>B</td>
<td>48.0</td>
<td>41,966,000</td>
<td>80.0</td>
</tr>
<tr>
<td>C</td>
<td>63.3</td>
<td>48,793,824</td>
<td>67.0</td>
</tr>
<tr>
<td>D</td>
<td>36.5</td>
<td>41,966,000</td>
<td>67.0</td>
</tr>
<tr>
<td>E</td>
<td>48.0</td>
<td>48,793,824</td>
<td>30.0</td>
</tr>
<tr>
<td>F</td>
<td>63.3</td>
<td>17,127,014</td>
<td>80.0</td>
</tr>
<tr>
<td>G</td>
<td>36.5</td>
<td>17,127,014</td>
<td>80.0</td>
</tr>
<tr>
<td>H</td>
<td>48.0</td>
<td>17,127,014</td>
<td>67.0</td>
</tr>
<tr>
<td>I</td>
<td>63.3</td>
<td>41,966,000</td>
<td>30.0</td>
</tr>
</tbody>
</table>

Table 6. Hypothetical family group preferences.

<table>
<thead>
<tr>
<th>Group</th>
<th>Hypothetical families</th>
<th>Preference structure</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A B C</td>
<td>C &gt; B &gt; (A)</td>
</tr>
<tr>
<td>2</td>
<td>D E F</td>
<td>E &gt; F &gt; D</td>
</tr>
<tr>
<td>3</td>
<td>G H I</td>
<td>G &gt; I &gt; H</td>
</tr>
</tbody>
</table>

and ranked three at a time for ease of comparison. Table 6 shows these groups and the resulting assumed preferences.

For example, hypothetical family C is preferred to hypothetical family B as less market coverage is sacrificed for a larger PCI and more profit. In total, nine trade-off scenarios are evaluated that give six preferences of one hypothetical family over another. As noted in Section 3.4, hypothetical family A is a dominated alternative and would never be realistically preferred by a rational decision maker. Therefore, any inequality constraint using hypothetical family A would not yield any useful information for the optimisation formulation. As a result, we disregard the preference B > A when forming inequality constraints in the next section.

3.6. Constructing the optimisation problem

The next step requires formulating the HEIM optimisation problem and its inequality constraints (Equation (1)) based on the five remaining preferences from Table 6. It is the solution of this problem that gives us a feasible set of attribute weights. It is worth noting that the HEIM optimisation problem particular to this stapler family consolidation problem uses only $x$ slack variables and inequality constraints in its formulation because the preferences in Table 6 are all inequivalents. Equivalent preferences would have resulted in equality constraints and subsequent $z$ slack variables in the formulation.

Using the $L_1$-norm value function from Equation (3) to develop all constraints, the optimisation problem takes the form shown in Equation (8):

Minimise: $x_{BC} + x_{FE} + x_{DF} + x_{IG} + x_{HI}$,

subject to: $g_1 : -0.5w_1 - 0.5w_2 + 0.5w_3 + x_{BC} \leq 0$,

$g_2 : 0.5w_1 - w_2 + w_3 + x_{FE} \leq 0$,

$g_3 : w_1 + 0.5w_2 - 0.5w_3 + x_{DF} \leq 0$. 

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The final step is the solution to the optimisation problems in Equations (8) and (11). As

are

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The simplified (and equivalent) versions of these constraints are shown in Equation (8). This
redundancy occurs solely because of the $L_1$-norm value formulation for the constraints in
Equations (9) and (10) and does not occur when the constraints are formulated using other norms. To
illustrate, a new optimisation problem formulation using the $L_2$-norm is shown in Equation (11),
where no redundancy exists in the constraints $g_1$ and $g_5$:

Minimise : $x_{BC} + x_{FE} + x_{DF} + x_{IG} + x_{HI}$

subject to : $g_1 : (0.5w_1 + 0.5w_2 + w_3) - (w_1 + w_2 + 0.5w_3) + x_{BC} \leq 0,$

$g_5 : (0.5w_1 + 0.5w_3) - (w_1 + 0.5w_3) + x_{HI} \leq 0.$

$w_1 + w_2 + w_3 = 1,$

$0 \leq w_1, w_2, w_3 \leq 1,$

$x_{BC}, x_{FE}, x_{DF}, x_{IG}, x_{HI} \geq 0.001.$

3.7. Solving the optimisation problem

The final step is the solution to the optimisation problems in Equations (8) and (11). As
discussed in Section 2.2, unless the optimisation problem is sufficiently constrained, many
equivalent sets of weights may exist that produce slack variables that minimise the objective
function and satisfy the constraints. Using a GRG-based method and setting a lower bound
of 0.001 on the slack variables, one of the non-unique solutions to Equation (8) is found
to be $(w_1, w_2, w_3)_{L_1} = (0.25, 0.60, 0.15)$ with slack variables $(x_{BC}, x_{FE}, x_{DF}, x_{IG}, x_{HI})_{L_1} = (0.001, 0.001, 0.001, 0.001, 0.001)$. Likewise, one of the non-unique solutions to Equation (11) is found to be $(w_1, w_2, w_3)_{L_2} = (0.35, 0.65, 0.00)$ with slack variables $(x_{BC}, x_{FE}, x_{DF}, x_{IG}, x_{HI})_{L_2} = (0.001, 0.001, 0.001, 0.001, 0.001).$
These attribute weights are then used to determine the overall value for each actual family of staplers (shown in Table 2). However, since multiple equivalent solutions exist for both value function formulations, in the next section, we study the impact of these multiple solutions and identify an optimal stapler family that is robust to multiple feasible weights under both formulations.

4. Value function study and stapler family selection

In this section, we explore the regions of equivalent solutions and the impact of the value function formulation on these regions. In Section 4.1, we compare the sets of solutions found using both value functions. Then in Section 4.2, we study specific sets of weights that are feasible for one but not for both optimisation problem formulations. In Section 4.3, we isolate one set of weights common to both formulations and select an optimal and robust stapler family configuration. Finally, in Section 4.4, we compare product family consolidation and selection using both HEIM and MAUT-based approaches and note important limitations.

4.1. Value function comparison

While both the $L_1$-norm and $L_2$-norm formulations have multiple equivalent solutions, some of the solutions for each formulation are not feasible for the other formulation. As an example, the two solutions given in Section 3.7 are both infeasible in the other formulation. To understand why this occurs, in Figure 5 we represent the three-dimensional design space for the $L_1$-norm optimisation formulation, which shows the three weights corresponding to the three attributes. The lightly shaded triangular region of Figure 5 shows all possible combinations of weights whose sum equals one. The smaller, darker shaded triangle in Figure 5 is the set of weights that satisfy the five constraints in the $L_1$-norm problem formulation (Equation (8)).

From the figure, it is clear that the redundant constraints, $g_1$ and $g_5$, do not actively bound the feasible region. The feasible sets of weights are determined exclusively from constraints $g_2$, $g_3$, and $g_4$. Combinations of weights in this region equivalently optimise the objective function of Equation (8) with $(x_{BC}, x_{FE}, x_{PF}, x_{IG}, x_{HI})_{L_1} = (0.001, 0.001, 0.001, 0.001, 0.001)$.

Similarly, the darker region in Figure 6 is the set of weights that satisfy the five constraints in the $L_2$-norm problem formulation (Equation (11)).

Figure 5. Feasible region under the $L_1$-norm.
Similarly in Figure 6, the feasible set of weights is determined only from constraints \(g_2\), \(g_3\), and \(g_4\). Even though constraints \(g_1\) and \(g_5\) are distinct, they still do not actively bound the feasible region. Combinations of weights in this region equivalently optimise the objective function of Equation (11) with \((x_{BC}, x_{FE}, x_{DF}, x_{IG}, x_{HI})_{L_2} = (0.001, 0.001, 0.001, 0.001, 0.001)\).

These figures help make sense of the results in Section 3.7, where there exist multiple solutions that are feasible for one formulation but not the other. Likewise, there are sets of solutions that are feasible (and optimal) for both formulations. In Table 7, the percentage of the larger triangular region (the \(w_1 + w_2 + w_3 = 1\) plane) that is feasible under each formulation is shown.

It is evident that a more extensive set of feasible weights is attained in the \(L_2\)-norm formulation. This is due to the fact that, under the \(L_1\)-norm, the linear constraint formulations are essentially tangent to the quadratic formulations and as a result bound a smaller feasible region. In the next section, we investigate these sources for dissimilar solutions across value functions in more detail.

### Table 7. Value function comparison.

<table>
<thead>
<tr>
<th>Value function constraint formulation</th>
<th>Percentage of unconstrained region (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(L_1)-norm</td>
<td>9.8</td>
</tr>
<tr>
<td>(L_2)-norm</td>
<td>14.6</td>
</tr>
</tbody>
</table>

4.2. **Dissimilar solution investigation**

In order to investigate these regions where the feasible solutions differ between the formulations, we isolate and correlate certain sets of weights to specific constraint violations. For instance, the shaded subset in Figure 7a represents the set of weights that satisfy the constraints formed using an \(L_1\)-norm but not an \(L_2\)-norm value function which is also visually apparent when comparing Figure 5 with Figure 6. Comprising 1.2% of the entire \(w_1 + w_2 + w_3 = 1\) plane, these solutions violate the \(g_3\) constraint in the \(L_2\)-norm formulation (Equation (8)). The multiple shaded subsets in Figure 7b represent the sets of weights that satisfy constraints formed using an \(L_2\)-norm but not an \(L_1\)-norm value function.

These regions of difference are also visually apparent when comparing Figure 5 with Figure 6. Specifically, the three shaded subsets shown in Figure 7b correlate to three particular constraint
violations under the $L_1$-norm. Collectively accounting for 6.0% of the entire $w_1 + w_2 + w_3 = 1$ plane, 27.6% of solutions in this subset violate the $g_2$ constraint, 31.2% violate the $g_3$ constraint and 41.2% violate the $g_4$ constraint from Equation (11).

In the next section, we use the feasible set of weights common to both value functions for product family selection. We contend that this common feasible region, comprising 8.6% of the entire $w_1 + w_2 + w_3 = 1$ plane, corresponds to the most accurate set of weights based on the decision maker’s preferences.

4.3. Product family selection

In this section, we identify an optimal stapler family using an extensive sampling of weights from the common feasible region. We develop different rankings of stapler families based on different sets of weights and demonstrate that although the stapler family ranking order is dependent on the selected weights, the stapler family with the highest value is robust for the entire common feasible region.

We sample combinations of weights from the common feasible region by varying $w_1$, $w_2$, and $w_3$ in increments of 0.01. In doing so, we generate 426 sets of feasible weights as shown in two dimensions in Figure 8, where the value for $w_3$ is found using $1 - w_1 - w_2$.

Overall values for each stapler family are calculated using each set of weights, and the family of stapler models 2, 3 and 4 is found to be optimal across all sets of feasible weights. The overall ranking of each stapler family is shown in Table 8 using both norms for two sets of feasible weights: $(w_1, w_2, w_3)_A = (0.33, 0.65, 0.02)$ and $(w_1, w_2, w_3)_B = (0.01, 0.50, 0.49)$. These sets correspond to two vertices of the common feasible region in Figure 8 and inherently represent two distinct trade-off mindsets as $(w_1, w_2, w_3)_A$ gives negligible importance to market coverage while $(w_1, w_2, w_3)_B$ gives negligible importance to PCI. These sets of weights are selected to illustrate the robustness of the solution across the wide range of feasible attribute weights.

The rankings in Table 8 support one robust solution, despite substantially different attribute weights. In fact, the top two families remain the same across both sets of weights and norms. The top optimal stapler family makes qualitative sense for a number of reasons. First, as evident in Table 2, this stapler family generates the greatest profit over five years. In addition, the preference structures from Table 6 confirm that short-term profit is valued much greater than PCI and market coverage. We would expect these preferences to surface in the final product rankings.

Also, as we see in Table 1, stapler models 1 and 2 are very similar in terms of performance. So as not to cannibalise each other in the market, we would expect an optimal family architecture to not include both of these models. In this case, the optimal stapler family included model 2,
which is the better performing stapler. Stapler models 3 and 4 both capitalise on demand in other segments of the market.

4.4. Method comparison and limitations

In Thevenot et al. (2006), the optimal product family configuration was also found to be stapler models 2, 3 and 4. This is primarily because both approaches use designer preferences and resulting attribute weights that favoured the greatest short-term profit. It is expected that if the preferences over the hypothetical alternatives were changed, a different product platform would result. In this section, we compare these methods and give insight into why, in certain circumstances, one method would be preferred over the other.

First, the HEIM-based approach may be favourable in terms of simplicity. This is due to the fact that evaluating attribute trade-offs and aggregating the utilities in MAUT can become quite challenging. For example, the first step in the MAUT process consists of ranking the attributes based on order of importance. This step alone can potentially cause problems for a decision maker as the order of attribute importance is not always obvious. In HEIM, however, a decision maker
need not know the order of attribute importance. Instead, that order is derived through a series of stated preferences over hypothetical product families.

In addition, when using MAUT, in order to determine the attribute scaling parameter $k_i$ values for an attribute $i$, a decision maker is required to evaluate various indifference points between alternatives. This is also a potential disadvantage because finding an exact indifference point is often a challenging and time-consuming process (Thurston 2001). When using HEIM, the decision maker is not required to specify an exact indifference point. Rather, the decision maker is asked only to determine whether two hypothetical alternatives are equivalent or not. Accordingly, HEIM, in this regard, may offer a less-challenging path towards determining a solution. Lastly, as presented in this paper, HEIM offers a way to study product family consolidation and selection in terms of robustness across multiple attribute weights. In Thevenot et al. (2006), the focus was not on robustness, but tactics to do so could include evaluating attribute scaling factors based on ranges of indifference points instead of choosing only one.

Still, a MAUT-based approach is an effective tool for product family consolidation and selection, as it not only quantifies a decision maker’s willingness to make trade-offs at different attribute levels but also integrates uncertainty in the process. When using MAUT, finding indifference points stems directly from one decision maker’s preferences or, in the case shown in Thevenot et al. (2006), a company’s business strategy. Furthermore, based on indifference points, MAUT only calls for simple mathematical calculations to determine each attribute scaling parameter. In the case of HEIM, an optimisation problem must be generated and solved perhaps more than once if multiple equivalent solutions exist. In addition, if the elicited preferences are inconsistent with each other, then the optimisation problem will have no feasible solution. This situation is addressed using an approach to ensure consistent preferences in Kulok and Lewis (2007).

Lastly, there are some limitations on the scalability of the HEIM approach as a function of the number of products, product family configurations and product attributes. As any of these problem parameters increase, the size of the resulting optimisation formulation increases. As the number of products increases, the number of possible family configurations increases combinatorially. For example, in a consolidation problem with 20 original products, the combinatorial possibilities for different product family configurations equal $1,048,575 \left[ C(20, 1) + C(20, 2) + \cdots + C(20, 20) \right]$. While HEIM will effectively eliminate dominated configurations, the number of constraints necessary to reduce the alternatives to a manageable number may be substantial.

An increase in the number of product attributes will result in an increase in the size of the experimental design required to sample hypothetical product family alternatives. More preference evaluations will be necessary, in turn increasing the number of constraints in the optimisation formulation. An increase in the number of attributes will also result in an increase in the number of design variables in the formulation, since the design variables are the attribute importances. When using MAUT, an increase in the number of product attributes may make it more difficult to rank attribute importances, and will also require more indifference evaluations.

5. Conclusions and future work

In this paper, we extend HEIM as a means to consolidate an existing product family of staplers and determine an optimal family architecture. To do so, we use two different value functions in an attempt to select attribute weights that best reflect a decision maker’s preferences. We explore why some sets of weights satisfy constraints formulated using one value function but not the other. Finally, we select an optimal stapler family that was robust across all common feasible combinations of attribute weights, and we compare this solution to that found using a MAUT-based approach. Although both product family consolidation methods yield the same solution, using HEIM helps to overcome some limitations of MAUT by allowing a decision maker to study
the robustness of the optimal solution. As global markets both collapse and develop, there are going to be more opportunities for product family consolidation and expansion. This approach provides effective decision support to designers as they increasingly use product platforms as key strategic components in new global product opportunities to generate revenue growth.

Future research includes studying the application of HEIM to a product family consolidation problem in which the solution is less defined. The stapler family consolidation problem explored in this work concluded that two staplers, one model that staples up to 15 pages and another model that staples up to 20 pages, compete with each other in the market, thereby decreasing company profits due to cannibalisation. While this conclusion is significant for validating the concept, the result is rather straightforward. Future research will focus on a less distinguishable product family consolidation problem that better represents the issues involved with supplying a greater number of products to a global marketplace.

A second area for future work concerns a study of the effects caused by selecting one certain product family. For example, by consolidating low-end products, a company may inadvertently allow a low-end competitor to move into the market and gradually take away market share from higher end products. This may affect the expected company profits and, ultimately, the entire strategy for product family configuration. Therefore, future work could focus on developing a feedback loop that gives insight into the behaviour of the competition caused by a certain choice in product family.

Finally, future work will aim to identify opportunities for extending HEIM to new product design. The research presented in this work only demonstrates HEIM as a reactive bottom–up approach for determining an optimal product family configuration, given that all products in a family already exist. Future research will study the possibilities for leveraging the same HEIM-based method in a proactive top–down manner for the design of new products or families of products. Consideration will have to be given to what a hypothetical alternative represents in the context of HEIM when the actual alternatives are still conceptual and therefore also hypothetical.

References


