Using Robust Design Techniques To Model The Effects Of Multiple Decision Makers In A Design Process

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ABSTRACT

In this paper we introduce a methodology to reduce the effects of uncertainty in the design of a complex engineering system involving multiple decision makers. We focus on the uncertainty that is created when a disciplinary designer or design team must try and predict or model the behavior of other disciplinary subsystems. The design of a complex system is performed by many different designers and teams, each of which only have control over a small portion of the entire system. Modeling the interaction among these decision makers and reducing the uncertainty caused by the lack of global control is the focus of this paper. We use well developed concepts from the field of game theory to describe the interactions taking place, and concepts from robust design to reduce the effects of one decision-maker on another. Response Surface Methodology (RSM) is also used to reduce the complexity of the interaction analysis while preserving behavior of the systems. The design of a passenger aircraft is used to illustrate the approach, and some encouraging results are discussed.

1 INTRODUCTION

In the design of a large, complex engineering system it is not uncommon for more than one design team to be involved. Often these teams are formed along disciplinary lines, each responsible for the design of a single part (subsystem) of the overall system. Quite possibly, each subsystem has its own goals and constraints that must be satisfied along with the system-level goals and constraints. In addition, the goals of the individual subsystems might be contradictory, that is, a satisfactory design from the view of one subsystem is not necessarily satisfactory from the view of the others.

In practice this problem may be overcome by iterating sequentially between the subsystems until a solution is obtained that is acceptable to all subsystems. Unfortunately, design iteration is both time consuming and expensive considering that in a large engineering system, (e.g. aircraft, automobile, etc.), there may be dozens of subsystems coupled together forming complex iterative loops.

Concurrent Engineering (CE) methods were developed to improve upon the sequential, ‘over the wall’ design commonly practiced. Multidisciplinary teams are formed involving engineers from every aspect of the product life cycle so that design goals and constraints from all of the subsystems can be considered and the appropriate tradeoffs made during the initial stages of the design process. Although the use of CE concepts can and has lead to increases in the efficiency of the design process, in some cases existing organizational structure within a design organization can hamper multidisciplinary interaction (Womack, et al., 1990). In addition, even when a concurrent approach is taken, some iteration may still take place due to communication or geographical barriers between design teams that are not collocated or which may not even be part of the same company (Lewis and Mistree, 1997a).

Clearly, it would be advantageous for a designer to be able to make a decision regarding the design of a subsystem independent of the other designers’ decisions. This problem provides the impetus for the method developed in this paper. We will explore a technique which allows for the design of a subsystem in the face of uncertainty stemming from incomplete information concerning the remainder of the system. We will accomplish this by approximating the information exchange between that subsystem and the remainder of the system. The proposed formulation is an integration of robust design principles with mathematically rigorous game theoretic models that together aid in describing the decision-making process and prescribing appropriate product decisions.

The remainder of the paper will be divided into three parts. First, we will introduce the relevant concepts from game theory, response surface approximations, and robust design that form the foundation of this work. Next, a general system design problem will be considered using the proposed formulation. Finally, in the final section we consider the design of a passenger aircraft to illustrate the practical application of this work.
2 BACKGROUND

In this section, the three primary foundations for this work are presented: game theory, response surfaces, and robust design.

2.1 Game Theory

Game theory is set of mathematical constructs that describe the interaction between multiple decision-makers. Although long a mainstay in the fields of economics and strategic warfare, it can be a powerful aid in the design process particularly when there is interaction between multiple designers, design teams, or companies (Hazelrigg, 1996, Lewis and Mistree, 1997b, Vincent, 1983).

Three game theoretic protocols have been developed to describe the interactions that occur among designers within a complex design process (Lewis and Mistree, 1997b). To avoid confusion it should be noted that in this context the term player (used extensively in game theory literature) is equivalent to designer or design team.

In the best case scenario, cooperation exists among the designers. Each designer has an exact representation of the information needed from the other designers. This type of decision-making environment leads to the highest quality solution and affords the designers with large amounts of design freedom (Pakala and Rao, 1996). Unfortunately, in practice perfect cooperation seldom occurs in the design of a large engineering system due to the complex interactions taking place both within and between design teams.

By contrast, the worst case scenario corresponds when designers are isolated. They act independently of one another without any exchange of information. This design information deficit necessitates a ‘design for worst case’ strategy where each team assumes the most unfavorable values for the needed design variables. As might be expected intuitively, noncooperation generally leads to a poor design and should be avoided (Rao, et al., 1997).

Between the two extremes of cooperative and isolated design environments is where most actual design processes lie. Designers share information when possible and make guesses when necessary. Also, the decisions made are often not completely concurrent in nature but occur sequentially. This may be due to long-standing practices within a company of one disciplinary team designing their subsystem first and passing this information on the second designer and so on. For instance in the design of a turbine engine, the design could start at the compressor subsystem, proceed to the combustor subsystem, and then be passed to the turbine subsystem. As the design process progresses each subsystem becomes increasingly constrained by the design decisions made by the preceding subsystems. However, since they have information regarding the previous design stages there is also a reduction in the level of uncertainty as the design moves towards completion.

Mathematically, the sequential or leader/follower design protocol can be expressed as a multi-level programming problem. For the sake of simplicity only the two-player case is considered. Player 1 (leader) has freedom to select values for the vector of design variables \( X_1 \). Player 2 (follower) has control over the vector of design variables \( X_2 \). In general, \( X \) contains both independent design and dependent state of behavior variables.

The problem from the standpoint of player 1 (leader) can be expressed as:

\[
\begin{align*}
\text{minimize} & \quad f_i(X_1, X_2) \\
\text{satisfy:} & \quad \text{goals and constraints} \\
X_2 & \in X_{\text{RRS}}(X_1)
\end{align*}
\]

where the last equation implies that \( X_2 \) is part of the Rational Reaction Set (RRS) of the follower, player 2. The Rational Reaction Set is a function that embodies the reaction of one player to decisions made by other players. As given by its name, it assumes that a player will act rationally. Therefore, in the formulation (1), the leader assumes that the follower will behave rationally, and this behavior is quantified in the leader’s RRS. The specific form of the RRS in this case is

\[
X_2 = f(X_1)
\]

and the general form of the RRS is given by:

\[
X_{\text{nonlocal}} = f(X_{\text{local}})
\]

where \( X_{\text{local}} \) is the vector of design variables in a single subsystem and \( X_{\text{nonlocal}} \) are the design variables in the remainder of the system.

Equation (2) implies that for a given vector \( X_1 \) in formulation (1), player 2 will choose the vector \( X_2 \) which minimizes their objective function (but the leader does not know the objective function of the follower, thus creating the possible conflict). The problem for player 2 (follower) becomes:

\[
\begin{align*}
\text{minimize} & \quad f_i(X_1, X_2) \\
\text{satisfy:} & \quad \text{goals and constraints} \\
\text{Given:} & \quad (X_1)
\end{align*}
\]

In other words, the follower now knows \( X_1 \) with certainty, but is constrained by it as well.

As the RRS is a function of one independent design variable in terms of other independent design variables, these functions are generally difficult or impossible to determine analytically. In the next section we will describe a method to approximate the RRS using Response Surface Methodology (RSM) and Design of Experiments (DOE).

2.2 Using RSM to Approximate the Rational Reaction Set

We have seen in the previous section the importance of the RRS for the solution of the sequential design protocol and the difficulty in determining it analytically. We have therefore chosen to use DOE and RSM to approximate these functions (Box and Draper, 1987). This approximation of the rational reaction sets represents a new bridging of existing concepts from game theory, statistics, and design of experiments. A similar recommendation on the use of statistics in design was made in (Simpson, et al., 1997). A conceptual outline for the construction of the Rational Reaction Sets is illustrated in Figure 1.
Two general categories of Robust Design (Chen, 1995). There are parameters than the traditional optimal design point. There are design is ‘less sensitive’ to variation in uncontrollable design uncertainty or variation (Phadke, 1989). In other words a robust minimizing the effect of uncertainty or variation in design objectives. Some techniques from Robust Design will be used to accomplish this goal.

In sensitivity while keeping the design objectives at near complete decoupling could result in an unsatisfactory design. It quality is generally sacrificed in favor of a more robust solution, subsystems could be completely decoupled, but, since design decisions made by competing subsystems as internal noise variables. The motivation is to reduce the interdependence of the subsystem thus reducing the effect of one designer’s decisions on the remainder of the designer’s. As discussed previously, complete decoupling of the subsystem variables would be ideal but unrealistic because generally speaking, a reduction in variation is accompanied by a reduction in the quality of design. Thus, a suitable tradeoff must be found between performance and robustness (Ramakrishnan and Rao, 1994).

The fundamental difference between what we are proposing and traditional Type I robust design lies in the definition of the noise factors. As opposed to external noise factors (ambient temperature, humidity) which by their very nature are uncontrollable (or prohibitively expensive to control), we are concerned with internal noise variables which are deterministic decisions made by the other designers, but not controllable or even known by everyone. The end goal is the same, however, namely, to minimize the influence of the noise on the subsystem under consideration.

Another important difference lies in the fact that we are not (although it would be possible to do so) trying to reduce the variation in the performance subsystem with respect to the noise variables, but instead to reduce the variation of the design variables of one subsystem to changes in another subsystem’s design variables.

With these differences new insight is needed into the problem, provided by the previously discussed concepts from game theory, specifically the Rational Reaction Set. The Rational Reaction Set maps the variables of one subsystem into another. We would like to find a relatively ‘flat’ area on this nonlinear hypersurface which would represent a point where there is minimum coupling between the subsystems.

3 PROBLEM FORMULATION

In Type I Robust Design, the goal is to minimize the variation caused by uncontrollable noise factors. Examples might include changes in ambient temperature, operating environment, or other natural phenomena that are impossible or prohibitively costly to control. In Type II Robust Design, the goal is to minimize variations caused by deviation in the control factors. This could result from manufacturing tolerance limitations or material variation (Sundaresan, et al., 1993).

In this paper, Type I Robust Design will be our primary focus as we seek to model unknown and uncontrollable design decisions made by competing subsystems as internal noise variables. The motivation is to reduce the interdependence of the subsystem thus reducing the effect of one designer’s decisions on the remainder of the designer’s. As discussed previously, complete decoupling of the subsystem variables would be ideal but unrealistic because generally speaking, a reduction in variation is accompanied by a reduction in the quality of design. Thus, a suitable tradeoff must be found between performance and robustness (Ramakrishnan and Rao, 1994).

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2.3 Robust Design Methodology

Fundamentally, robust design is concerned with minimizing the effect of uncertainty or variation in design parameters on a design without eliminating the source of the uncertainty or variation (Phadke, 1989). In other words a robust design is ‘less sensitive’ to variation in uncontrollable design parameters than the traditional optimal design point. There are two general categories of Robust Design (Chen, 1995).
Given
An alternative to be improved through modification. Assumptions used to model the domain of interest.

Find
Design variables
Deviation variables, \( d_i^- \), \( d_i^+ \)

Satisfy
System constraints (linear, nonlinear)
System goals (linear, nonlinear)
\[
A_i(x) + d_i^- - d_i^+ = G_i; \quad i = 1, \ldots, m
\]
Design Variables Bounds

Minimize
Deviation function
\[
Z = [f_1(d_1^-, d_1^+), \ldots, f_k(d_k^-, d_k^+)]
\]

Figure 2: General Form of Compromise DSP

The basic form of the two player problem can be expressed in terms of two coupled Decision Support Problems as shown in Figure 3. Player 1 (P1) has control over the vector of design variables \( x_1 \) in subsystem 1 while Player 2 (P2) controls the vector of design variables \( x_2 \) in subsystem 2. Due to the coupling in the subsystems, a portion of the total number of design variables in each subsystem is needed by the other subsystem. Thus, the two vectors \( x_1 \) and \( x_2 \) can be subdivided into the coupled and uncoupled parts. We define \((x_{c1}, x_{nc1})\) and \((x_{c2}, x_{nc2})\) as the coupled and uncoupled portions of \( x_1 \) and \( x_2 \) respectively. Thus, the goals and constraints for P1 are functions of \((x_{c1}, x_{nc1})\) while the goals and constraints for P2 are a function of \((x_{c2}, x_{nc2})\). In general design problems, there is also coupling of state or behavior variables. This approach incorporates these couplings as well, however, in this paper, it is not the focus.

![Figure 3: Decision Support Models for Player 1 and 2](image)

P1 has two types of goals: performance and robust goals. The performance goals consist of the original, system dependent goals (e.g. minimize total weight, cost, etc.). The robust goal (as explained in section 2.3) tries to force the solution to a point in the design space where P2’s design vector \( x_{c2} \) is relatively insensitive to P1’s design vector, \( x_{c1} \). This is accomplished by reducing the gradient of P2’s Rational Reaction Set to a prescribed level. The gradient is determined by analytically differentiating the second order approximation to the RRS (section 2.2).

A problem associated with using the gradient it that it is only valid for small deviations around the point of interest. In cases where this assumption does not hold, it would be advantageous to use a measure of robustness that is valid over a wider range. Previous attempts to measure robustness estimated the variance of a response (Chen, 1995). We found that this may cause numerical stability problems in the optimization routine, as the variance can be orders of magnitude larger than the response itself. Also, in the Taguchi model of robustness, the measure of robustness is of similar units to the response itself (Taguchi, 1987). By using the gradient to measure the robustness, the value of the response at a given design point can be multiplied by the response itself to get a feel for the range of deviations in the same units as the response. This issue is under investigation currently but not addressed in this paper. Related issues and comparisons of different measures of robustness are discussed in (Su and Renaud, 1996).

Although some information must indeed be passed from one designer to another in order to construct the Rational Reaction sets, the amount is directly dependent on the amount of coupling between the two subsystems, as only the Rational Reaction Sets for the design variable vectors \( x_{c1} \) and \( x_{c2} \) must be formulated. This vector could contain only a small fraction of the total number of design variables for that subsystem.

Using the Rational Reaction set we can introduce robustness into the design by looking for ‘flat’ areas in the design space defined by the RRS. Player 1 would like to desensitize the vector of design variables \( x_{c1} \) to changes in the value of Player 2’s vector of design variables \( x_{c2} \).

It is possible for the leader, P1, for instance, through the choice of \( x_{c1} \), to choose an region in the design space where the variation of \( x_{c2} \) with respect to \( x_{c1} \) is minimized. This would benefit the overall system because during subsequent design changes, changes in P1 would have minimum effect on P2. This represents a departure from the classical leader/follower scenario because we have introduced a small amount of cooperation. This cooperation has the tendency to worsen the position of the leader for the sake of the follower and robustness of the overall system. This occurs because a robust solution is seldom the optimal solution. Therefore, our goal is to use approximations of the RRS interactions between subsystems to determine robust solution regions where the behavior of each subsystem is relatively insensitive to changes in other subsystems while not significantly sacrificing performance quality.

4 CASE STUDY: DESIGN OF A PASSENGER AIRCRAFT

To illustrate the usefulness of the approach we will consider the design of a passenger aircraft previously considered in (Lewis, 1996). The system is subdivided into two subsystems, an aerodynamics player and a weight player. The Decision Support Problems for each are in Figure 5 below. It should be noted that these are the original formulations before the robust goals are integrated.
Variables needed by aero designer:
Take-off Weight, Wto
Installed Thrust, Ti

Variables needed by weights designer:
Wing Area, S

Figure 5: Aerodynamics and Weights Compromise DSPs

It can be seen that there are a total of 5 total design variables, 3 controlled by Player Aero (X_aero: Wing Area - S, Fuselage Length - l, Wing Span - b), and 2 controlled by Player Weight (X_weight: Take-off Weight - Wto, Installed Thrust - Ti). Player Aero needs both Wto and Ti from the Weight player, but Player Weight only needs S from the Aero player. In this study, there are also five state variables coupled between the disciplines and accounted for in the Rational Reaction Sets. However, the results focus on the values and effects of the coupled independent design variables. In the next sections, we investigate 2 cases: Player Aero as Leader and Player Weight as Leader. We also compare these two cases and make some observations about the system in general and the significance of design process structure on the quality of the final solution.

4.1 Case 1: Player Weight is Leader
Let us consider the case where there exists a sequential design situation with Player Weight as leader and Player Aero as follower. As discussed above, in order to find a robust solution, the leader needs to locate a flat region in the Rational Reaction Set of Player Aero which is assumed to be known. Since a second order response surface is used to calculate the Rational Reaction Set it is possible to plot the surface $S=f(WtoN,TiN)$ where the N indicates the variables were normalized between -1 and 1. The gradient S is given by

$$\nabla S = \frac{\partial S}{\partial WtoN} \frac{\partial S}{\partial TiN}$$

Plots of both S and $\nabla S$ are given below in Figures 6a and b.

Figure 6a: S vs. Wto and Ti

Figure 6b: $\nabla S$ vs. Wto and Ti
From Figure 6a it seems as though the ‘flattest’ region occurs at the lower bounds of WtoN and TiN. This is confirmed by Figure 6b in which the gradient reaches is lowest value at (WtoN=-1, TiN=-1). Also, it is clear that the gradient is much more dependent on WtoN then on TiN as evidenced by the rapid change in gradient when moving from the upper to the lower bound of WtoN. The first step was to determine a non-robust baseline solution for comparison purposes. The original compromise DSP’s (Figure 5) were solved with player weight as leader and player aero as follower using the protocols from (Lewis and Mistree, 1997a). The robust goals were then added to form the robust weight compromise DSP which is given below in Figure 7 (the aero compromise DSP remains the same as in the nonrobust model). Then, the robust formulation of the leader/follower protocol was solved using different priority levels and weightings of the robust and performance goals. The resulting solutions (design variables, deviation function, and RRS gradient) are shown in Table 1.

The top half of Table 1 are the leader (Weight) solutions and the lower half are the various solutions for the Aero follower using the Leader’s solution. The first solution is the non-robust leader/follower solution. The remaining 9 solutions are the various solutions corresponding to the robust formulations. For solution 1, the nonrobust solution, the deviation function is the smallest. Solutions 2 through 10 represent an increasing importance of robustness, respectively. For each of these 10 solutions, the values of Ti and Wto are then passed to the aero model and that problem is solved. The design variables of the aero model are fuselage length (l), wing area (S), and wingspan (b).

There are a number of insights that can be made from Table 1. First, recall that only S, the wing area, is coupled to the Weight design problem. Because of this, in the top half of Table 1, the Weight player is using an approximated prediction of the Wing Area, S (how the Aero player will react). This prediction is denoted by S(RRS). In the lower half of the table, the actual values of S after the Aero player solves their model are given. It can be seen that the predicted S(RRS) values range from 1660 to 1556 ft² while the actual S values range from 2127 to 2098 ft². This discrepancy is attributed to using only a second order approximation of the wing area. Although the R² value of the second order response surface of S as a function of Wto and Ti is 0.949, a higher order model might be warranted if more accurate results were needed. It is important to note that the predicted S(RRS) and the actual S will be the same if the approximation of S is perfect. The prediction of the variables of the follower is a risk that the leader takes: they are predicting how the follower will react using a probabilistic approximation. The disciplines making their decisions later in a design process frequently do not follow what the previous disciplines had envisioned. However, quantifying with some certainty the rational reaction set gives designers more information to make more effective decisions than having no information predicting the effects of coupled decisions.

<table>
<thead>
<tr>
<th>Design #</th>
<th>Ti(lbs)</th>
<th>Wto(lbs)</th>
<th>S(RRS) (ft²)</th>
<th>∇S</th>
<th>DevFunc</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (non-robust)</td>
<td>40627.6</td>
<td>205402</td>
<td>1619.31</td>
<td>397.383</td>
<td>0.203</td>
</tr>
<tr>
<td>2</td>
<td>40480.8</td>
<td>200092</td>
<td>1566.82</td>
<td>374.642</td>
<td>0.203</td>
</tr>
<tr>
<td>3</td>
<td>41899.1</td>
<td>210712</td>
<td>1660.39</td>
<td>423.087</td>
<td>0.206</td>
</tr>
<tr>
<td>4</td>
<td>38195.5</td>
<td>197639</td>
<td>1575.52</td>
<td>348.901</td>
<td>0.208</td>
</tr>
<tr>
<td>5</td>
<td>37144.1</td>
<td>196197</td>
<td>1571.52</td>
<td>348.901</td>
<td>0.215</td>
</tr>
<tr>
<td>6</td>
<td>36140.1</td>
<td>195000</td>
<td>1574.3</td>
<td>341.02</td>
<td>0.225</td>
</tr>
<tr>
<td>7</td>
<td>36140.1</td>
<td>195000</td>
<td>1574.3</td>
<td>341.02</td>
<td>0.222</td>
</tr>
<tr>
<td>8</td>
<td>36251.7</td>
<td>196377</td>
<td>1586.6</td>
<td>347.159</td>
<td>0.224</td>
</tr>
<tr>
<td>9</td>
<td>35697</td>
<td>194536</td>
<td>1576.41</td>
<td>337.81</td>
<td>0.225</td>
</tr>
</tbody>
</table>

Table 1: Weight as Leader Results
Aero Robust Compromise DSP

Given
- Follower’s RRS, \( W_{to} = f(S) \)
- \( T_i = f(S) \)

Find
- Design variables
- Deviation variables, \( d_i^- , d_i^+ \)

Satisfy
- System constraints
- Performance Goals

Robust Goals:
- \( \nabla W_{to} = \left[ \frac{\partial W_{to}}{\partial S} \right] \) (N - normalized)
- \( \nabla T_i = \left[ \frac{\partial T_i}{\partial S} \right] \) (N - normalized)

Minimize
- Deviation function

Figure 10: Robust Aero Compromise DSP (Aero as Leader)

Second, in the nonrobust solution (Solution 1), the gradient of \( S \) (which, again, represents the effect of the follower on the leader) is very high. As can be seen in solutions 2 through 10, the deviation function increases in these solutions, but the gradient decreases. When the robust goal is the most important (Solution 10), the robustness of the solution increases by 15%, while the deviation function increases by 10%. This tradeoff between performance and robustness is shown clearly in Figure 8 where the various Weight solutions from Table 1 are plotted. There is a clear trend in the regression that suggests that at design points where the gradient of \( S \) is smaller, the deviation function is larger.

A third point to make is that the nonrobust solution and the most robust solution, in this example, are similar. Therefore, the nonrobust solution happens to also be a fairly robust solution. However, this is only for this problem and is a system dependent characteristic. In the next set of results, we consider the reverse of the sequence we considered in this case.

4.2 Case 2: Player Aero as Leader

Now we will consider the case where the Player Aero acts as leader and Player Weight as follower. As in the previous case the goal of the leader is to find robust regions in the RRS of the Player Weight. One difference between this and the previous case lies in the number of variables that are needed by the leader. In the case 1, only \( S \) was needed by the leader, but in case 2, both \( W_{to} \) and \( T_i \) are needed. Another difference is that in case 1, \( S \) was dependent on both \( W_{to} \) and \( T_i \), whereas in case 2, the Rational Reaction Sets constructed for \( W_{to}, T_i \) are dependent only on one variable, \( S \).

The Rational Reaction Sets along with their gradients are plotted below in Figure 9. Once again, the RRS’s are quadratic (as expected from the use of the second order response surface). An interesting difference between this case and case 1 lies in the fact that \( \nabla W_{to} \) and \( \nabla T_i \) pass through zero (become flat at some point). Thus, if a feasible solution was found in an area of the design space around these points, the sensitivity of Player Weight’s design to that of Player Aero would be at a minimum.

Figure 8. Design Quality vs. Robustness, Case 1

Figure 9: RRS Interactions
- a. \( W_{to} \) vs. \( S \) and \( \nabla W_{to} \) vs. \( S \)
- b. \( T_i \) vs. \( S \) and \( \nabla T_i \) vs. \( S \)
As in Case 1, the nonrobust problem was initially solved to determine baseline values for both the gradients and the deviation. We then moved on the robust model given below in Figure 10.

The remaining 5 solutions are the various solutions corresponding to the robust formulations. For solution 1, the nonrobust solution, the deviation function is the smallest. Solutions 2 through 6 represent an increasing importance of robustness, respectively. For each of these 5 solutions, the values of S were then passed to the weight model and their problem was solved. The design variables of the weight model were take-off weight (Wto) and installed thrust (Ti), and both of these variables are needed in the aero problem. The lower half of Table 2 shows the various solutions for the weight follower using the Leader’s solution.

<table>
<thead>
<tr>
<th>Aero Player (Leader)</th>
<th>Design #</th>
<th>b(ft)</th>
<th>S(ft²)</th>
<th>l(ft)</th>
<th>( \nabla \text{Wto} )</th>
<th>( \nabla \text{Ti} )</th>
<th>Dev Func</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (non-robust)</td>
<td>138.841</td>
<td>1986.64</td>
<td>132.23</td>
<td>13244.5</td>
<td>-1297.99</td>
<td>0.252</td>
<td></td>
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<tr>
<td>2</td>
<td>139.529</td>
<td>2091.9</td>
<td>135.87</td>
<td>5929</td>
<td>-2740</td>
<td>0.27</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>95.54</td>
<td>1831.32</td>
<td>126.853</td>
<td>7302.42</td>
<td>489.36</td>
<td>0.334</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>95.09</td>
<td>1797.38</td>
<td>125.6</td>
<td>9557.05</td>
<td>950.67</td>
<td>0.308</td>
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<tr>
<td>5</td>
<td>97.14</td>
<td>1853.15</td>
<td>127.61</td>
<td>6361.56</td>
<td>196.774</td>
<td>0.332</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Weight Player (Follower)</th>
<th>Ti (lbs.)</th>
<th>Wto (lbs.)</th>
<th>Dev Func</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (non-robust)</td>
<td>34175</td>
<td>193204</td>
<td>0.286</td>
</tr>
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Table 2: Weight as Leader Results

There are again a number of similar insights that can be made from Table 2. First, the predicted and actual Wto and Ti values (not shown) in this case were very similar, unlike Case 1. The \( R^2 \) values for Wto and Ti as a function of S are 0.900 and 0.892. Therefore, the second order response surfaces were adequate in this case for predicting nonlocal behavior.

Second, in the nonrobust solution (Solution 1), the gradients of Wto and Ti (which, again, represents the effect of the follower on the leader) are relatively high. As can be seen in solutions 2 through 6, the deviation function increases in these solutions, but the gradient decreases. When the robust goal is the most important (Solution 10), the robustness of the solution increases by 50% with respect to changes in Wto, and 85% with respect to changes in Ti, while the deviation function increases by 30%. This tradeoff between performance and robustness is shown clearly in Figure 11 where the gradients of Wto and Ti are plotted vs. the normalized deviation functions for the various Aero solutions from Table 2. There is a clear trend in the regression that suggests that at design points where the gradients of Wto and Ti are smaller, the deviation function is larger. This trend is similar to case 1 with Weights as the leader, only accentuated even more with greater increases in robustness while also sacrificing more in the performance of the system.

Another insight into this case, is the fact that the deviation function for the follower, Player Weight, remains relatively constant for each leader solution, implying that the addition of robustness has little if any effect on the quality the followers solution. This is a desired result, as even though the two problems are coupled, the changes made in the Aero solution are not effecting the Weight solution. In other words, the Weight solution is robust to the Aero solution. This was not observed as clearly in Case 1, and is problem dependent.

4.3 Overall Results

We have made observations concerning the quality of the solution for each player individually. However, engineers, designers, and managers are also concerned about how well the system performs as a whole. Therefore, if we examine the overall quality of the solutions by simply adding the individual
deviation functions together, we can find an interesting result. When the models are solved without robustness, the solution of Case 2, when the aero player is leader, is better. However, when robustness is the primary priority, the solution of Case 1, when weights is leader, is better. On the other hand, solutions with more robustness (less sensitive to changes in the other player) are found when Aero is leader. So, there are larger tradeoff issues in this problem as well. This lends insight into managing and sequencing a design process depending upon the freedom afforded by each design team as a result of decisions made upstream in a process. In this case, either sequence, Case 1 or Case 2, could be preferred depending upon the overall system requirements and preferences of the designers involved.

5 CLOSURE

We have introduced a design methodology to mitigate the effects of uncertainty in design processes involving multiple decision makers. Using game theory as a mathematical tool to describe the actions of decision makers with incomplete information, we have applied concepts from robust design traditionally applied to systems with a single decision maker. In essence, the goal was to decouple the subsystems and reduce the sensitivity of one player’s decisions to decisions made by other players. Preliminary results have shown that is indeed possible to make a tradeoff between robustness and performance quality. Thus, in a situation where the cost of uncertainty is high, these tools can be used to increase the robustness, or independence of the subsystems. Designers can then make more effective decisions without having to worry about the effects on other subsystems, without having to guess at unknown information, and without having to perform numerous iterations to converge with coupled disciplines. This work represents a step towards achieving these objectives. Further investigations will include increasing the level of approximation of the interactions between designers, application to a jet turbine engine in continued work with Allison/Rolls Royce Engine Company, and increasing the number of players involved in the design process.

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REFERENCES


