A Comprehensive Robust Design Approach for Decision Trade-Offs in Complex Systems Design

In this paper we introduce a technique to reduce the effects of uncertainty and incorporate flexibility in the design of complex engineering systems involving multiple decision-makers. We focus on the uncertainty that is created when a disciplinary designer or design team must try to predict or model the behavior of other disciplinary subsystems. The design of a complex system is performed by many different designers and design teams, each of which may only have control over a portion of the total set of design variables. Modeling the interaction among these decision-makers and reducing the effect caused by lack of global control by any one designer is the focus of this paper. We use concepts from robust design to reduce the effects of decisions made during the design of one subsystem on the performance of the rest of the system. Thus, in a situation where the cost of uncertainty is high, these tools can be used to increase the robustness, or independence, of the subsystems, enabling designers to make more effective decisions. To demonstrate the usefulness of this approach, we consider a case study involving the design of a passenger aircraft. [DOI: 10.1115/1.1334596]

Keywords: Robust Design, Multidisciplinary Design, Optimization, Game Theory

1 Introduction

In the design of a large, complex engineering system it is not uncommon for more than one design team to be involved. Often these teams are formed along disciplinary lines, each responsible for the design of a single part (subsystem) of the overall system. Quite possibly, each subsystem has its own goals and constraints that must be satisfied along with the system-level goals and constraints. In addition, the goals of the individual subsystems might be contradictory, i.e., a satisfactory design from the view of one subsystem is not necessarily satisfactory from the view of the others. Finally, the subsystems are often coupled, i.e., there are design variables in the disciplinary sub-problems controlled by other disciplinary designers.

Concurrent Engineering (CE) methods were developed to improve upon the sequential, ‘over the wall’ design commonly practiced. Multidisciplinary teams are formed involving engineers from every aspect of the product life cycle so that design goals and constraints from all of the subsystems can be considered and the appropriate tradeoffs made during the initial stages of the design process. Although the use of CE concepts can and has lead to increases in the efficiency of the design process, in some cases existing organizational structure within a design organization can hamper multidisciplinary interaction [1]. In addition, even when a concurrent approach is taken, some iteration may still take place due to communication or geographical barriers between design teams that are not collocated or which may not even be part of the same company [2].

Clearly, it would be advantageous for a designer to be able to make a decision regarding the design of a subsystem independent of the other designers’ decisions. As a senior Honda executive recently remarked, “We wish we could design, engineer, fabricate, and assemble the entire car in one large room, so that everyone involved could be in face-to-face contact with everyone else.” [1]. This problem provides the impetus for the method developed in this paper. We explore a technique that allows for the design of one part of the system in the face of uncertainty stemming from incomplete information concerning the remainder of the system. We accomplish this by approximating unknown design information as noise variables and use robust design techniques to mitigate their effect. The proposed formulation is an integration of robust design principles with models of the design process that together aid in understanding the decision-making process and prescribing appropriate product decisions.

Although pursuing concurrency is the ideal in many situations [3] other design scenarios are better suited to sequential models. Modeling the subsystems sequentially results in a less complex model as compared to concurrent models. In addition, a sequential model more realistically represents many actual design processes. In a sequential model, decisions made by the designers at the beginning have a strong influence on the design decisions made downstream. Consider a system consisting of multiple subsystems executed sequentially. Design decisions made by the first designer may make it difficult for the remaining designers downstream to find satisfactory or even feasible designs. To help mitigate this problem, research is ongoing to develop mathematical models of the design process and techniques that allow designers to solve their disciplinary sub-problems independent of the rest of the system [4,5].

One such technique developed by Chen and Lewis [6] integrates game theoretic models of the design process with robust design techniques in order to reduce the effect of the coupling between the subsystems and improve the performance of the overall system. In this technique the initial designer chooses a range of acceptable design variable values as opposed to a point solution. The proceeding designers then have the freedom to select from any values within this range, yielding improved performance. Although this technique is quite effective, in cases where there is coupling in both directions, (the designers downstream in the design process need information from the designers upstream, and the upstream designers need design information from the downstream designers) it becomes necessary to estimate or approximate the latter. These estimates are often of the worst case variety and while previous work has shown that approximating the unknown information is effective [7,2], it may require considerable computational effort and some coordination among the designers.

Monu Kalsi
Kurt Hacker
Kemper Lewis*

Department of Mechanical and Aerospace Engineering, State University of New York at Buffalo Clifford C. Furnace Hall, Box 604400 Buffalo, NY 14260-4400

*Corresponding author.
Contributed by the Design Automation Committee for publication in the JOURNAL OF MECHANICAL DESIGN. Manuscript received Nov. 1999. Associate Editor: A. Diaz.
Our approach addresses these same issues but in a different manner. We also seek to find a region of the design space where the performance is robust to changes in the control factors, but instead of guessing at or approximating the unknown design information, we treat it as a noise variable and use robust design techniques to mitigate its effect. We seek a solution that is not only robust with respect to changes in the initial designer’s control variables but also stable with respect to the unknown design variables controlled by the downstream designers.

It is important to point out that the approach presented in this paper is comprehensive in the sense that both Type I and Type II robust design are used to increase decision-making robustness. It is not meant to convey the idea that all design scenarios are robust to unknown design information. Instead, we treat it as a noise variable and use robust design techniques to mitigate its effect. We seek a solution that is not only robust with respect to changes in the control factors, but also robust with respect to the uncertainty or variation in the initial designer’s control variables.

2 Technical Background

2.1 Mathematical Models of Multidisciplinary Design. A typical model of a multidisciplinary optimization model is given in Fig. 1 [5,6]. The coupling in the system is represented by the linking variables, \( y_{ij} \), which are the design or behavior variables that are needed by discipline \( j \) and determined during the analysis of discipline \( i \). The symbols \( f \) and \( g \) represent subsystem objectives and constraints, respectively.

Since these linking variables are essential in the solution to the disciplinary sub-problems, the mechanism by which their values are communicated between the disciplinary designers is of great importance. There are a number of ways in which a designer might approach the problem of communicating or representing linking variables.

1 Ignore Uncertainty

A guess is made for the uncertain linking variables.

2 Seek Out Perfect Information

The disciplinary designers determine the linking variables by collaboration with the other designers.

3 Represent and Manipulate the Uncertainty

Techniques such as robust design can be used to make the solution robust to the uncertainty in the linking variables.

Intuition tells us that collaboration between the designers should yield the best results, but in many cases there may be barriers which make full cooperation difficult or impossible [8]. Communication between the interacting disciplinary subsystems might be hampered by geographical separation (i.e., teams may not be colocated) or by the fact that the design teams are part of different departments within the same company or even different companies. Guessing at the unknown design information is always an option, especially when based on experience. But if the guess is far off, the result is degradation in performance and expensive and time-consuming iteration. Modeling the interaction between the designers and making the solution robust to the uncertainty may be the preferred strategy in many cases.

2.2 Robust Design. Fundamentally, robust design is concerned with minimizing the effect of uncertainty or variation in design parameters (variables and constants that appear in a design problem formulation) without eliminating the source of the uncertainty or variation [9]. In other words a robust design is ‘less sensitive’ to variation in uncontrollable design parameters than the traditional optimal design point. Robust design has found many successful applications in engineering and is continually being expanded to different design phases [10–12]. There are two general categories of robust design [13].

In Type I robust design, the goal is to minimize the variation caused by uncontrollable noise factors. Examples might include changes in ambient temperature, operating environment, or other natural phenomena that are impossible or prohibitively costly to control.

In Type II robust design, the goal is to minimize variations caused by deviation in the control factors (i.e., design variables). This could result from manufacturing tolerance limitations, material quality variations, or even evolving design preferences [14]. Figure 2 illustrates this. In this figure the variation in performance \( y(x) \) for a traditional ‘optimal’ design and a robust design are compared when the design variable \( x \) varies a quantity \( \Delta x \) about its mean value \( \mu \). Although robust design has been traditionally applied in manufacturing there has been research recently into applying these techniques to make the design conceptually robust [13,15]. The important roles of modeling and calculation of robustness in a multidisciplinary design environment is discussed in [16]. Our work builds upon the philosophy of these references - we are trying to make design decisions robust to uncertainty caused by evolving design goals and constraints in a multidisciplinary design environment.

The approach we are presenting is an integration of Type I and Type II robust design. Type I is used to make the leader’s solution robust to unknown design decisions made by the follower. Recall that the leader in a sequential design process must solve a disciplinary sub-problem under uncertainty, not knowing how the follower will act. The fundamental difference between what we are proposing and traditional Type I robust design lies in the defini-
tion of the noise factors. As opposed to external noise factors (ambient temperature, humidity) which by their very nature are uncontrollable (or prohibitively expensive to control), we are concerned with internal noise variables, deterministic decisions made by the other designers, but not controllable or even known by everyone. The end goal is the same, however, to minimize the influence of uncertainty on the subsystem under consideration.

We are also incorporating Type II robust design to introduce flexibility into the design. We use the approach developed in Chen and Lewis [6], where the idea of a robust solution range is used. The first designer chooses a range of satisfactory designs instead of a traditional point solution. In this way, Designer 1 is allowing the subsequent designers more freedom to find a satisfactory solution to their disciplinary sub-problems. Chen’s formulation is presented in Eq. (1). The goals are to bring the mean of the performance on target and minimize the variation about the mean through the proper selection of x and Δx. It should be noted that Eq. (1) is only exact when the constraints gj(x) are linear. It is a good approximation, however, when Δx is small. This form is used in this work because it avoids the computational expense that would be incurred if other methods were used to determine whether the constraints are satisfied.

Find: x, Δx
Minimize: [μj, σj]
subject to gj(x) + kj ∑ ∂gj ∂xi Δxi ≤ 0, j = 1, 2, ..., J
xL ≤ x ≤ u, Δx ≤ u − x, Δx

This technique is effective for problems where there is strictly one-way coupling between the subsystems, specifically in problems where the follower needs design information from the leader, but the leader needs no design information from the follower. In many systems, however, there is two-way coupling in which both the leader and the follower need design information (design or behavior variables) from the other. Chen and Lewis [6] overcome this problem by assuming that the designers downstream react rationally to the design decisions made upstream of them. This reaction is embodied in a construct from game theory called the Rational Reaction Set [17,18]. When this function is known, the leader in a sequential design process can predict what the reaction of the follower will be to a design decision, allowing the leader’s disciplinary sub-problem to be solved [2]. Since the RRS is a function of one independent design variable in terms of another independent design variable, an analytical expression for it is impossible to determine for all but the simplest cases. Instead a combination of Design of Experiments and Response Surface Methodology is used to approximate it. Design points are sampled from the upstream designer’s design space and are used as input to the downstream designer’s subsystem model. An optimization is performed for every set of input points and a response surface is fitted to these points. The resulting function closely approximates the Rational Reaction Set [7].

The difficulty in this approach lies both in the actual construction of the Rational Reaction Set, which may be computationally expensive due to the number of optimization runs needed, and the fact that its use necessitates interaction among the designers. As this may not always be possible in real design situations, we propose to instead model this unknown interaction as an additional noise parameter and use Type I robust design techniques to mitigate its effect [13,15]. This will be accomplished by finding values of the local design variables where the performance variation lies within a tolerable range subject to variation in the downstream subsystems linking parameters, y21. The only prior information needed is the ranges for the downstream subsystem design variables, which we assume to be known. This approach will be presented in detail in Section 3.

<table>
<thead>
<tr>
<th>Designer 1 Model</th>
<th>Designer 2 Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Given</td>
<td>Given</td>
</tr>
<tr>
<td>Bounds on y_{12}</td>
<td>Bounds on y_{12}</td>
</tr>
<tr>
<td>Find</td>
<td>Find</td>
</tr>
<tr>
<td>Control Variables x_i</td>
<td>Control Variables x_j</td>
</tr>
<tr>
<td>Deviation Variables d_i', d_j'</td>
<td>Deviation Variables d_i', d_j'</td>
</tr>
<tr>
<td>Satisfy</td>
<td>Satisfy</td>
</tr>
<tr>
<td>Constraints g(x_i y_{12}) ≤ 0</td>
<td>Constraints g(x_j y_{12}) ≤ 0</td>
</tr>
<tr>
<td>Goals</td>
<td>Goals</td>
</tr>
<tr>
<td>A_{ij}(x_i y_{12}) - d_i' + d_j' = G_0</td>
<td>A_{ij}(x_j y_{12}) - d_i' + d_j' = G_0</td>
</tr>
<tr>
<td>Bounds on x_i</td>
<td>Bounds on x_j</td>
</tr>
<tr>
<td>Minimize</td>
<td>Minimize</td>
</tr>
<tr>
<td>Deviation Function Z_2 = f(d_i', d_j'), ..., f(d_i', d_j')</td>
<td>Deviation Function Z_2 = f(d_i', d_j'), ..., f(d_i', d_j')</td>
</tr>
</tbody>
</table>

Fig. 3 Compromise DSP formulations for leader/follower design protocol

3 An Approach for Conceptual Robust Design

Consider the system in Fig. 3. It is composed of two disciplinary subsystems: Designer 1 controls design vector x_1 and Designer 2 controls design vector x_2. We use the compromise Decision Support Problem (DSP) to model the system [19]. The compromise DSP includes concepts from mathematical and goal programming. The objective is to minimize the deviation from the target goal values established by the designer while satisfying system constraints. We have chosen this formulation because it allows the modeling of the design goals separately and on different priority levels, providing insight into the tradeoff between performance and robustness.

Consider the case in which the design process is sequential in nature with Designer 1 as the leader and Designer 2 as the follower. The value of the linking variables, y_{12}, need to be determined before a solution to Subsystem 1 can be determined. The first option in such a case is for Designer 1 to guess values for the linking variables, y_{12}. A strategy might be adopted to minimize the maximum value of the objective function. In other words, Designer 1 would assume that Designer 2 is going to choose the worst possible values of y_{12}. This invariably leads to less than optimal results for both subsystems, assuming a feasible solution can be found at all. Another possibility is to assume that Designer 2 will behave rationally. As described in Section 2.2, Designer 2 constructs an analytical function of the form y_{12} = f(x_1), which allows a solution to Subsystem 1 to be found. This is the standard Stackelberg/Leader-Follower approach [2].

The approach that is presented in this paper provides an additional option to handling the uncertainty. The unknown linking variables, y_{12}, needed by Designer 1 are modeled as noise variables with uniform probability distributions varying within modified bounds that lie within the actual bounds of the linking variables. The modified bounds are the maximum ranges of these linking variables within their total bounds that guarantee a robust and feasible local solution for Subsystem 1. This idea is illustrated in Fig. 4, which shows the true and modified bounds. Designer 1 selects these bounds by dividing the design space of the non-local linking variables y_{12} into a number of ranges within which y_{12} is uniformly distributed. Designer 1 then optimizes his model for performance and robustness for each of these ranges of y_{12}. As-

![Fig. 4 Modeling y_{12} with a uniform distribution](Image)
Designers 1 - Leader  
**Given:**  
Bounds on $y_{21}$  
**Find:**  
$[y_{11}, y_{21}, \text{lower}, y_{21}, \text{upper}]$  
**Satisfy:**  
Constraints:  
$g_i(x_1, y_{21}) = \sum \frac{\partial g_i}{\partial y_{21}} y_{21} \leq 0$  
Goals:  
$A_i(x_1, y_{21}) - d_i^* + d_i^* = G_i$  
$
\left( \frac{\partial A_i}{\partial y_{21}} \right) \sigma_{\Delta y_{21}}^2 + d_i^* = 0
$
**Bounds:**  
$x_{\text{lower}} \leq x_1 \leq x_{\text{upper}}$  
**Minimize:**  
$Z_1 = f(d_i^*, d_i^*) - f(d_i^*, d_i^*)$  

Designers 2 - Follower  
**Given:**  
$[y_{11}, y_{21}, \text{lower}, y_{21}, \text{upper}]$  
**Find:**  
$y_{11}$  
**Satisfy:**  
Constraints:  
$g_i(x_1, y_{21}) = \sum \frac{\partial g_i}{\partial y_{21}} y_{21} \leq 0$  
Goals:  
$A_i(x_1, y_{21}) - d_i^* + d_i^* = G_i$  
**Bounds:**  
$x_{\text{lower}} \leq x_1 \leq x_{\text{upper}}$  
**Minimize:**  
$Z_2 = f(d_i^*, d_i^*) - f(d_i^*, d_i^*)$  

![Fig. 5 Compromise DSP for aerodynamics as leader/weights as follower with Type I robust design consideration](image)

though this requires running an optimization for each range it allows a good region of the design space to be identified for further analysis. The largest range possible that allows Designer 1 to find a feasible and satisfactory solution is chosen. The upper and lower bound to this range is then passed to Designer 2. Note that there is a minimum bound on this range (range cannot collapse to zero) to ensure that a robust region is found.

A uniform distribution is used to model the linking variables by making the assumption that the other designer has an equal probability of choosing any value of the unknown variables within those bounds. If the designers are not communicating, this assumption is reasonable. Two parameters, mean and variance, given in Eqs. (2) and (3) describe a uniform distribution.

$$\mu_{y_{21}} = \frac{(y_{21,U} + y_{21,L})}{2} \quad (2)$$

$$\sigma_{y_{21}}^2 = \frac{(y_{21,U} - y_{21,L})^2}{12} \quad (3)$$

We introduce two new design parameters into the model for Subsystem 1, $\mu_{y_{21}}$ and $\Delta y_{21}$. These parameters describe the location of the modified bounds $y_{21,U}$ and $y_{21,L}$ within the true bounds $y_{21,U}$ and $y_{21,L}$. The goals for Subsystem 1 can be divided into two categories; goals related to performance and those related to robustness or uncertainty. Expressions for the mean and variance of the performance are given in Eqs. (4) and (5). Equation 5 is exact only when $f_1$ is linear and approximates the true variance for nonlinear objective functions. This approximation is used to avoid the considerable expense of determining the variance using another method such as Monte Carlo analysis.

Mean of the Performance:  
$$\mu_{f_1} = f(x_1, \mu_{y_{21}}) \quad (4)$$

Variance of the Performance:  
$$\sigma_{f_1}^2 = \left( \frac{df_1}{d\mu_{y_{21}}} \right) \sigma_{y_{21}}^2 \quad (5)$$

In the modified problem, the objective is to bring the mean of the performance on target and minimize the variance of the performance about the mean. These goals can be placed on different priority levels based on the relative importance of performance or robustness. If performance is given priority, the variance may be relatively high, while conversely, if robustness is given priority the mean of the performance may not be on target. To ensure feasibility of the constraints we assume that the noise or control factors of the system vary simultaneously in the worst possible combinations [14]. Figure 5 contains the compromise DSPS showing the flow of information between the subsystems.

From the perspective of Subsystem 2, this formulation additionally constrains the feasible design space to be within the modified bounds, $y_{21,U}$ and $y_{21,L}$, which are chosen by Designer 1 and are likely to be more restrictive than the true bounds. In addition, Designer 1 also dictates $x_1$, which may further constrain Subsystem 2. To help mitigate the effect these additional constraints have on the performance of Subsystem 2 and introduce flexibility into the design process, we include Chen’s [6] approach for conceptual robustness described in Section 2.2. With the added flexibility of being able to select from a range of possible values for the linking variables as opposed to a point solution, Designer 2 is more likely to be able to find a design that is feasible and satisfies performance goals. This range is selected such that, within it, the objective of Subsystem 1 is stable with respect to changes in $x_1$. Hence, the performance of Subsystem 1 will likely be worse than at the point solution because the robust solution is often different from the standard optimal solution.

Despite this drawback, reducing the downstream coupling in a preliminary design environment has important implications. As design goals and constraints evolve during the design process large amounts of redesign might become necessary if the performance of one subsystem is tightly coupled and sensitive to changes in the other subsystems’ design variables. If the uncertainty present is not addressed significant amounts of time and effort might be wasted in the preliminary stages of the design process through costly design iteration. The approach we have presented in this section has the potential to reduce the impact of uncertainty in the preliminary stages of design and provide the designer with a starting point for more detailed design. In the next section, we illustrate the usefulness of this approach by considering the preliminary design of a passenger aircraft.

4 Case Study: Design of a Passenger Aircraft

4.1 Compromise Decision Support Problem Models. To demonstrate our approach we consider the design of a passenger aircraft previously considered in [7,6]. The system is partitioned into two subsystems, an aerodynamics subsystem and a weight subsystem. The Compromise Decision Support Problem (DSP) for each is shown in Eqs. (6) and (7). For each constraint and goal, functional notation is given, showing the variables that are required from the other designer. As is evident, the variables Take-off Weight, $W_{to}$, and Installed Thrust, $T_i$, are needed from the Weight designer by the Aero designer and the variable, Wing Area, $S$, is needed from the Aero designer by the Weight designer.

The formulations shown for each designer are without any robust design considerations. The aerodynamics subsystem has three control variables; wing span ($b$), fuselage length ($l$) and wing area ($S$). The Weights subsystem has two control variables, take-off weight ($W_{to}$) and installed thrust ($T_i$). The models were developed largely based on [20]. The full aerodynamic and weight models are given in the Appendix. The reader is directed to [7] for more details on the derivation of certain relationships and assumptions in the model.

**Find**

<table>
<thead>
<tr>
<th>The values of the system variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Units</td>
</tr>
<tr>
<td>Wing area, $S$</td>
</tr>
<tr>
<td>Fuselage length, $l$</td>
</tr>
<tr>
<td>Wing span, $b$</td>
</tr>
</tbody>
</table>

**Satisfy**

The system constraints

\begin{align*}
\text{Aspect ratio} & \leq 10.5 \\
\text{Achievable climb gradient on landing} f(W_{to}, T_i) & \geq 2.4^\circ \\
\text{Achievable climb gradient on take-off} f(W_{to}, T_i) & \geq 2.4^\circ \\
\text{Landing field length} f(W_{to}) & \leq 4,500 \text{ ft.} \\
\text{Take-off field length} f(W_{to}, T_i) & = 6,500 \text{ ft.} \\
\text{Drag coefficient in take-off and landing} & f(W_{to}) \leq 0.02 \\
\text{The drag coefficient in cruise} & f(W_{to}) = 0.02 \\
\end{align*}

The values of the deviation variables associated with the goals
The system goals, i.e., 1.5
Missed Approach Climb Gradient, landing f(Wto, Ti)
Climb Gradient, take-off f(Wto, Ti)
Landing Field Length f(Wto)
Take-off Field Length f(Wto, Ti)
Aspect Ratio

The bounds on the system variables

Find

The values of the system variables
Take-off weight, Wto [lb]
Installed thrust, Ti [lb]

The values of the deviation variables associated with the goals

Satisfy

The system constraints
Useful load fraction f(S) > 0.3
Fuel available/fuel required f(S) ≥ 1.0
Landing field length f(S) ≤ 4,500 ft.
Take-off field length f(S) ≤ 6,500 ft.
Drag coefficient in take-off and landing f(S) ≤ 0.02
The drag coefficient in cruise f(S) = 0.02
The system goals, j = 1, 6
Useful load fraction f(S)
Climb Gradient, take-off f(S)
Landing Field Length f(S)
Take-off Field Length f(S)

Minimize

The sum of the deviation variables

Z = \Sigma w_j (d_j^- + d_j^+) 

WEIGHTS MODEL

In Section 4.2, we investigate the case in which the Aerodynamics subsystem is the leader and the Weights subsystem is the follower in a sequential design process. In other work, we investigate the reverse case where Weights is the leader [21], but we focus on the case where the Aerodynamics is the leader in this paper. We consider three scenarios. In Scenario I, a baseline solution is found without any robust design considerations using the Rational Reaction Set to determine the value of the unknown linking variables, Wto and Ti. In Scenario II, Wto and Ti are modeled as noise variables with uniform distributions. In Scenario III, we combine Scenario II with Chen’s [6] robust design approach to increase decision flexibility for the follower, the Weights subsystem. All formulations are solved using the DSIDES software, which utilizes an augmented sequential linear programming approach [19].

4.2 Robust Design Approach to Minimize Control and Noise Factor Deviation. Scenario I: Without any robust design considerations, the design problem can be modeled using the leader/follower design protocol, with the unknown linking variables determined using the Rational Reaction Set (RRS) introduced in Section 2.2. In this case we use the Rational Reaction Set to determine Wto and Ti (y2j) for any value of S (x1) [6]. For comparison purposes, the designs obtained using the RRS will be used as a baseline. The values of the design variables and the performance deviation function for this scenario are presented in Table 1. The deviation function is indicative of the amount the design differs from the target values of the goals. The objective is to minimize its value.

Table 1 Solutions of Aero as leader/weights as follower model for Scenario I

<table>
<thead>
<tr>
<th>AERO (LEADER)</th>
<th>WEIGHT (FOLLOWER)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design Variables</td>
<td>Wto (lbs)</td>
</tr>
<tr>
<td>B (ft)</td>
<td>131.1</td>
</tr>
<tr>
<td>S (ft)</td>
<td>1853</td>
</tr>
<tr>
<td>L (ft)</td>
<td>146.4</td>
</tr>
</tbody>
</table>

Table 2 True and modified bounds for Ti and Wto

<table>
<thead>
<tr>
<th>True Bounds</th>
<th>Modified Bounds</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wto</td>
<td>Ti</td>
</tr>
<tr>
<td>140000</td>
<td>Ti</td>
</tr>
<tr>
<td>174800</td>
<td>Wto</td>
</tr>
</tbody>
</table>

Scenario II: In this scenario, the aerodynamics compromise DSP is modified based on robust design principles as described in Section 3. In addition to finding the value of S in the Aerodynamics subsystem model, we also find a maximum range of the nonlocal coupling variables Wto and Ti such that the Aero solution is robust and feasible within that range. We run a preliminary pattern search to find the most promising range in the Weight subsystem’s design space and then refine our analysis to maximize the modified bounds on Wto and Ti. In the preliminary search the weight subsystem’s design space is divided up into a grid of equal size sections. For each section the center point is the nominal value of Wto and Ti, and the size of the grid section in the Wto and Ti directions determines ΔWto and ΔTi. An optimization is performed to determine if a feasible solution can be found using these modified bounds. In the final analysis the size of the selected grid section is then maximized to increase the amount of coupling. There is a minimum bound on the grid size to ensure having a nonzero range. As a result, the solution found may not be optimal, as performance is sacrificed for reduced coupling. The Weights subsystem is then constrained to find a solution within the modified bounds passed to him by the leader.

Three different design cases are considered when solving Scenarios II and III:

- Case A: An Archimedean formulation is used in which the goals for both robustness and performance are placed at the same level with equal weights.

- Case B: A preemptive formulation is used with the emphasis on performance (performance goals are placed at the top priority level).

- Case C: A preemptive formulation is used with the emphasis on robustness (robustness goals are placed at the top priority level) [22].

The values of the modified bounds of the coupling variables Wto and Ti are found to be approximately 16 percent and 12 percent respectively of the true variable ranges. The true and modified ranges are shown in Table 2. These are the largest values modified ranges that allow the Aerodynamics discipline to still find acceptable solutions. Moreover, beyond this, the Taylor series approximations of the constraints may become invalid. The results of the optimization, consisting of the design variables, linking variables, and deviation functions for the Aero subsystem are presented in Table 3. The maximum standard deviation allowed in the performance is limited to 15 percent of the mean of the performance. The corresponding solution for the Weights subsystem when the Aero subsystem is the leader is shown in Table 4.

As observed from the results obtained in the three cases the deviation function values for both the Aerodynamics and Weights subsystems increase as compared to the solution obtained in Scenario I when there were no robust design considerations. The effect of this increase in the deviation function depends on the preference that exists between achieving a good solution and eliminating downstream coupling as explained in Section 3. We explore this trade-off further in Scenario III.

Scenario III: In addition to the Type I robust design introduced in the previous scenario, we want to provide the follower with increased flexibility in their design decisions. Since the follower is
constrained to choose designs within the modified ranges passed to him by the leader, the ability to choose from a range of acceptable linking variable values \( \Delta S \) instead of a single design point \( S \), increases the chances of finding an acceptable solution. The formulation of the compromise DSP for Aero is the same as the previous scenario, except for the range of wing area, \( \Delta S \), is now an additional design variable in the Aerodynamics subsystem model. The constraint and variance equations are augmented to include the effect of \( \Delta S \). The compromise DSP for Weights is modified so that, in addition to his own design variables, the Weights subsystem chooses the best value of \( S \) from the range \( \Delta S \) passed to him by the Aerodynamics subsystem. The modified bounds of the linking variables \( W_{t0} \) and \( T_{i} \) are the same as those given in Table 2. The results consisting of the values found for the design variables, linking variables, and deviation functions for the Aerodynamics subsystem are presented in Table 5. This solution is subsequently passed to the Weight subsystem along with the range of the linking variable \( \Delta S \). The solutions of the Weights subsystem are shown in Table 6.

In all three cases the performance of the Weights subsystem has improved as compared to Scenario II while the performance of the Aerodynamics subsystem has worsened. We have set the lower bound of \( \Delta S \) to 150, which is approximately 10 percent of the total range of variable \( S \). Depending on the leader’s willingness to sacrifice his performance and the need for achieving flexibility this minimum range value could be increased [4].

As a summary, plots of results for three cases of different priority levels are shown in Fig. 6. For the Aerodynamics model, we plot the performance variance against the performance mean deviation for Scenarios I, II, and III labeled ‘w/o RD,’ ‘Type I RD,’ and ‘Type I & II RD,’ respectively. Note that to give a more complete overview of the tradeoffs involved between performance and robustness we have also included results for Type II Robust Design from [6] labeled Type II RD in Fig. 6. The Weights subsystem performance deviation function is also plotted to observe the response to the Aerodynamics subsystem behavior.

It can be seen from plots a, c and e that the deviation function for Aerodynamics is the smallest in w/o RD (Scenario I), which is expected because the linking variables do not vary. Including Type II robust design only improves Weight subsystem’s solution as seen in plots b, d and f, and the leader has to sacrifice his performance to some extent. With Type I robust design considerations (Scenario II), both the Aerodynamics and Weights subsystem performances are sacrificed. This sacrifice is offset by a reduction in the coupling effects between the subsystems, making the designs less sensitive to the inevitable changes in the objectives and constraints that take place as the design evolves through more detailed stages. Combining Type I and II robust design (Scenario III) provides the benefits of both scenarios and allows the leader to find a solution insensitive to the coupling variables while the follower is provided with decision flexibility. What scenario should be followed depends on the design objectives and the designers’ willingness to sacrifice performance and gain robustness.

Investigating the individual and total system deviations for the four scenarios, the following conclusions could be made about the factors affecting the outcome of the decision process:

• Individual and system level performance depends upon the decision sequence chosen by the designers. In this paper, we only present detailed results from the aerodynamics as leader/weight as follower sequence. We also explored the reverse sequence where the weight discipline is leader. To illustrate the impact of decision sequence, Fig. 7 compares the total system deviation for weight as leader and that for aerodynamics as leader for the case when the performance and robustness goals are on equal priority level. This total system deviation is the sum of the deviation functions for the weight and aerodynamics subsystems. In Fig. 7, the overall system performance is better with Aerodynamics as leader for all scenarios. This is not always the case, however. Figure 8 shows the same scenarios when robustness is given priority over performance. Here it is clear that the system level performance is better when weight is leader [21]. Therefore, depending upon the goals of a design process—performance, robustness, or a combination—a different sequence of the disciplinary decision-makers should be used.

• The performance of the designs depends upon the scenario chosen. This depends upon the designers’ preferences for system performance and robustness. Depending upon the designers’ willingness to gain decision robustness, flexibility or both, the appropriate scenario can be chosen. Also, depending upon the designers’ preferences, different priorities could be given to the goals using other weighting schemes or placing the objectives at differ-

<table>
<thead>
<tr>
<th>Design Variables</th>
<th>Case A</th>
<th>Case B</th>
<th>Case C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ti (lbs.)</td>
<td>390.62</td>
<td>385.50</td>
<td>388.98</td>
</tr>
<tr>
<td>Wto (lbs.)</td>
<td>193.067</td>
<td>193.200</td>
<td>193.200</td>
</tr>
<tr>
<td>Linking S (ft)</td>
<td>167.8</td>
<td>151.1</td>
<td>249.8</td>
</tr>
<tr>
<td>Deviation Function</td>
<td>1.46</td>
<td>1.76</td>
<td>1.87</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Design Variables</th>
<th>Case A</th>
<th>Case B</th>
<th>Case C</th>
</tr>
</thead>
<tbody>
<tr>
<td>b (ft)</td>
<td>108.7</td>
<td>109.1</td>
<td>111.4</td>
</tr>
<tr>
<td>S (ft²)</td>
<td>1619</td>
<td>1629</td>
<td>2322</td>
</tr>
<tr>
<td>ΔS (ft²)</td>
<td>152.1</td>
<td>169.1</td>
<td>171.6</td>
</tr>
<tr>
<td>L (ft)</td>
<td>135.1</td>
<td>132.6</td>
<td>106.1</td>
</tr>
<tr>
<td>Deviation Function</td>
<td>1.61</td>
<td>4.32E-02</td>
<td>1.63</td>
</tr>
</tbody>
</table>
ent priority levels. We have used the Archimedean (with equal weights) and preemptive approaches (upper level objectives are assumed to be infinitely more important than lower level objectives) in this work, both of which are limited in finding and populating Pareto solutions. To fully explore the tradeoffs among robustness, flexibility, and performance, other multiobjective techniques such as [23–26] could be employed.

### Table 6  Solutions of weights as follower for Scenario III

<table>
<thead>
<tr>
<th>Design Variables</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tf (lbs)</td>
<td>39079</td>
<td>38650</td>
<td>38898</td>
</tr>
<tr>
<td>W0 (lbs)</td>
<td>193095</td>
<td>193200</td>
<td>193200</td>
</tr>
<tr>
<td>Linking S (ft)</td>
<td>1771</td>
<td>1798</td>
<td>2150</td>
</tr>
<tr>
<td>Deviation Function</td>
<td>1.44</td>
<td>1.63</td>
<td>1.66</td>
</tr>
</tbody>
</table>

Fig. 6  Comparison of Aero and weights subsystem deviation functions

---

**a. Aerodynamics as Leader: Equal Priority**

**b. Weights as Follower**

**c. Aerodynamics as Leader: Performance-Level I**

Robustness Level II

**d. Weights as Follower**

**e. Aerodynamics as Leader: Robustness-Level I**

Performance Level II

**f. Weights as Follower**
The solution outcome in Scenario III (Type I and II robust design) depends on the number of ranges being exchanged, the size of the ranges and also the portion of the design variable vector that needs to be passed as a range. For example, there is not much increase in the deviation of the Aerodynamics subsystem in Scenario III when a range of solutions $D_{\text{so}}$ is passed to the Weight subsystem. There is, however, a large increase in the deviation function of the Weight subsystem when Weight is leader and therefore has to pass ranges $D_{\text{Wt}}$ and $D_{\text{Ti}}$. This is primarily because the Aero designer as leader only passes ranges for $1/3$ of his total design variable vector and consequently still has freedom to control the fuselage length, $l$, and wing span, $b$. When Weight is leader, both design variables are passed to the Aero designer as ranges, constraining the solutions to small areas of the design space where the solutions are stable.

One of our assumptions in this work is that the subsystem models are distinct and cannot be combined. If the models could be combined, better solutions could be found [2]. However, in this work, we acknowledge model coupling and attempt to model and handle the coupling to increase flexibility and robustness in the decision. By finding regions where subsystem decisions are insensitive to those in other subsystems, many times performance is sacrificed. We have witnessed this in the example presented. Therefore, it is dependent on the preferences of the decision makers to determine whether performance can be sacrificed for robustness or flexibility and if so, how drastic or subtle this sacrifice can be.

### 5 Closure

In this paper, we develop a comprehensive design methodology to incorporate robust design concepts into multidisciplinary design. It is comprehensive in that sense that both types of robust design are considered. We apply Type I robust design for internal noise reduction and combine that with Type II robust design to achieve a combined robust and flexible design.

This approach is developed to reduce downstream coupling in an evolving design environment making the subsystems upstream less likely to be affected by changes in the linking variables of the downstream subsystems. By minimizing the effects of the decisions made by one discipline upon the others, we feel that time spent in iteration can be reduced, while improving the designers’ ability to make decisions concurrently. We feel that the application of robust design techniques to minimize the effect of non-local design information can prove extremely beneficial to complex systems design.

### Acknowledgments

We would like to acknowledge the support from NSF Grants DMI-9800435, DMI-9709942 and NASA Fellowship Grant NGT-152185. We are also grateful to Dr. Wei Chen, University of Illinois at Chicago for her support and advice.

#### Aerodynamics Subsystems Model

**Constant Aerodynamic Parameters**

- Aircraft maximum lift coefficient, $c_{L_{\text{max}}}$: 2.6
- Number of engines, $N$: 3.0
- Airfoil thickness-to-chord ratio, $t/c$: 0.12
- Number of passengers, $N_p$: 188
- Engine specific fuel consumption, $b_t$: 0.0019444 lb/lb-sec
- Range, $R$: 2900 nmi, $1.762 \times 10^7$ feet
- Density, sea-level (take-off, landing), $\rho_{\text{t1}}$: 0.002378 slugs/ft$^3$
- Kinematic viscosity, sea-level (take-off, landing), $\nu_{\text{t1}}$: 0.000156 ft$^2$/sec
- Kinematic viscosity, 35,000 ft (cruise), $\nu_c$: 0.000406 ft$^2$/sec
- Velocity, sea level (take-off and landing), $V$: 220 ft/sec
- Density, 35,000 ft (cruise), $\rho_c$: 0.000737 slugs/ft$^3$

#### Important Aerodynamic Relationships and Equations

Zero lift drag coefficient

$$c_{DO} = (C_{DO})_{\text{wing}} + (C_{DO})_{\text{body}} + \Delta C_{DO}$$

Wing contribution

$$c_{DO}^{\text{wing}} = 1.1 c_{f,\text{wing}} \left( 1 + 1.2 \left( \frac{f}{c} \right)^4 + 100 \left( \frac{f}{c} \right)^4 S_s \right)$$

Body contribution

$$c_{DO}^{\text{body}} = c_{f,\text{body}} \left( 1 + 0.0025 \left( \frac{l}{d} \right) + 100 \left( \frac{l}{d} \right)^3 S_s \right)$$
Skin friction coefficient
\[ c_f = 0.455 \left( \log_{10} \left( \frac{V_{ref} l_{ref}}{V} \right) \right)^{-2.58} \]

Velocity
\[ V_{ref} = V, \text{ if in take-off or landing} \]
\[ V_{br}, \text{ if in cruise} \]

Reference length
\[ l_{ref} = \begin{cases} 1, & \text{for body} \\ c, & \text{for wing,} \ c = S/b \end{cases} \]

Body wetted surface ratio
\[ S_S = \frac{\pi d l}{S} \left( 1 - 2 \frac{d}{l} \right)^{2/3} \left( 1 + \left( \frac{d}{l} \right)^{2} \right) \]

Incremental drag coefficient
\[ \Delta C_{D0} = 0.005 \]

Fuselage diameter
\[ d = 1.83 \left( 4.325 \frac{N_p}{l} + 1 \right) \]

Oswald factor
\[ e = 0.96[1 - (d/b)^2] \]

Quadratic drag polar
\[ k = \frac{1}{[\pi (AR)e]} \]

Lift-to-Drag, landing and take-off
\[ L \frac{D}{D_{cruise}} = \frac{c_L}{c_{D0} + k c_L^2} \]

Lift Coefficient, take-off and landing
\[ C_L = \begin{cases} \frac{2W_{TO}}{\rho V^2 S}, & \text{if in take-off} \\ \frac{2W_{TO}(1-R_{sl})}{\rho V^2 S}, & \text{if in landing} \end{cases} \]

Lift-to-Drag, best cruise ratio
\[ \frac{L}{D_{cruise}} = \frac{1}{2} \left( 1 - e^{-\frac{1}{\sqrt{c_{D0cruise}}}} \right) \]

Velocity for best range
\[ V_{br} = \sqrt{\frac{2W_{TO}}{\rho S \sqrt{\frac{c_{D0}}{k}}}} \]

Landing Field Length
\[ S_L = 400 + \frac{118*W_{TO}(1-R_{sl})}{(c_{Lmax} S)} \]

Take-off Field Length
\[ S_{TO} = \frac{20.9*W_{TO}^2}{(c_{Lmax} S^3 T_{i})} + 87 \sqrt{\frac{W_{TO}}{c_{Lmax} S}} \]

Missed Approach Achievable Climb gradient, OEI, landing
\[ q_L = \left( \frac{L}{D} \right)_{L}^{-1} + \frac{N-1}{N} \frac{T_i}{W_{TO}(1-R_{sl})} \]

Achievable climb gradient, OEI, take-off
\[ q_{TO} = -\left( \frac{L}{D} \right)_{TO}^{-1} + \frac{N-1}{N} \frac{T_i}{W_{TO}} \]

Aspect ratio
\[ AR = b^2 \frac{S}{D} \]

Aerodynamic goals
Missed Approach Climb Gradient, landing
\[ q_L/0.03 + d_2 - d_1 = 1 \]

Climb Gradient, take-off
\[ q_{TO}/0.03 + d_2 - d_1 = 1 \]

Landing Field Length
\[ S_L/4500 + d_1 - d_2 = 1 \]

Take-off Field Length
\[ S_{TO}/4500 + d_1 - d_2 = 1 \]

Aspect Ratio
\[ AR/10.5 + d_1 - d_2 = 1 \]

The bounds on the aerodynamics variables
- Wing area (ft.): 1200 ≪ S ≪ 2500
- Fuselage length (ft.): 105 ≪ l ≪ 150
- Fuselage diameter (ft.): 85 ≪ d ≪ 140

Weight Subsystem Model

Constant Weight Parameters
- Aircraft maximum lift coefficient, c_{Lmax}: 2.6
- Number of engines, N: 3.0
- Engine specific fuel consumption, b_t: 0.00019444 lb/lb-sec
- Range, R: 2900 nmi, 1.762 × 10^7 feet
- Payload (cargo and passengers), W_{pay}: 40,000 lbs
- Fixed equipment weight, W_{fix}: 1100 lbs

Important Weight Relationships and Equations

Fuel Weight Available
\[ R_{fa} = 1 - \frac{W_{pay}}{W_{TO}} - \frac{W_{fix}}{W_{TO}} - \frac{W_{empty}}{W_{TO}} \]

Empty Weight Ratio
\[ W_{empty} \frac{W_{TO}}{W_{fix}} = 0.9592 + 0.38 T_i^{0.9881} \]

Fuel Weight required for mission
\[ R_f = 1.1 \left( 1 - 0.95 \frac{T_i}{W_{f}} \right) \]

Overall Fuel Balance
\[ R_f = \frac{R_{fa}}{R_{f}} \]

Ratio of take-off weight to landing weight
\[ W_{f} \frac{W_{TO}}{W_{f}} = \exp \left( - \frac{R_{b_f}}{V_{cruise}} \frac{L}{T_{cruise}} \right) \]

Useful Load Fraction
\[ U = 1.1(1 - 0.95 e^{-b_f \cdot R_b \cdot L \cdot V_{cruise} \cdot T_{cruise}}) + \frac{W_{pay}}{W_{TO}} \]
Weight goals

Useful Load Fraction
\[ \frac{U}{0.5 + d_i^1 - d_i^2} = 1 \]

Fuel Balance
\[ R_i + d_i^2 - d_i^3 = 1 \]

Missed Approach Climb Gradient, landing
\[ q_{\text{TOO}}/0.03 + d_i^5 - d_i^6 = 1 \]

Climb Gradient, take-off
\[ d_{\text{TOO}}/4500 + d_i^6 - d_i^7 = 1 \]

Landing Field Length
\[ S_f/4500 + d_i^7 - d_i^8 = 1 \]

Take-off Field Length
\[ S_{\text{TO}}/4500 + d_i^8 - d_i^9 = 1 \]

The bounds on the weight variables

Installed thrust (lbs.) \[ 27,750 \leq T_i \leq 55,000 \]

Take-off weight (lbs.) \[ 140,000 \leq W_{\text{TO}} \leq 250,000 \]

Nomenclature

- \( x_i \): Design variables for each discipline \( i \)
- \( y_{ji} \): Linking variables that are evaluated by discipline \( i \)
- \( f_i \): Objective functions of discipline \( i \)
- \( g_i \): Constraints of discipline \( i \)
- \( \mu_f \): Mean of the objective function \( f \)
- \( \sigma_f \): Standard deviation of the objective function \( f \)
- \( \Delta_x \): Deviation range of design solution
- \( b \): Wing span, ft.
- \( l \): Fuselage length, ft.
- \( S \): Wing area, ft.\(^2\)
- \( W_{\text{TO}} \): Takeoff weight, lbs.
- \( T_i \): Installed thrust, lbs.

References